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This is the author's original submitted version of the review. The finally accepted version of record appeared in the *British Journal for the History of Mathematics,* 2020, 35:2, 173-175, https://doi.org/10.1080/26375451.2020.1735618

Philip Ording, *99 Variations on a Proof*, Princeton University Press, 2019, 272pp, £2200

Philip Ording's *99 Variations on a Proof* presents the reader with ninety-nine different presentations and interpretations of a proof of a simple theorem about the solutions to a particular cubic equation with a repeated root. The book is written to echo Raymond Queneau's *Exercises in Style*[1] from 1947, in which Queneau repeatedly retells the same short story of an altercation on a bus in different literary styles to explore the effects, limits and entanglement of style and storytelling. Ording sets out to parallel this with an exploration of mathematical style.

Ording presents the proof in styles such as "elementary", "one-line", "axiomatic", "matrices", "clever" and "intuitionist". More historical variations include two "medieval" proofs, one from "antiquity", and a "found" proof from Cardano's *Ars Magna*[2], though historical snippets are found scattered throughout the book. To push the limits of mathematical style, there are also a number of weird, unusual and ornate renderings of the proof, which are best left for the reader to discover themselves (though my personal favourite is the "Mondegreen" proof).

Put simply: this book is marvellous and is to be recommended to everyone with even a passing interest in mathematics. For the professional mathematician, the variations should be humorously familiar, full of curious titbits about the history and practice of mathematics, and raising philosophical questions about the way mathematics is done. For the interested non-expert, the book provides a timely and rare insight into the heterogeneity of mathematics, and tells a rich story of the surprising diversity of styles of mathematics. Rather than the caricature of mathematics as dry and impenetrable, the book displays mathematics as a lively, exciting and growing discipline with a wealth of history.

To go along with each of the ninety-nine theorem-proof variations, Ording provides brief background commentaries through which the author gives us historical, social and mathematical context for each variation. These commentaries are also where the author's own voice shines through, making the adventure through mathematical styles seem both joyful and surprising.

The quality of the physical book is also to be commended, comparable to the wonderful recent Taschen edition of Oliver Byrne's colourful recreation of the first six books of Euclid's *Elements*[3] from 1847. I highly recommend that readers get a physical copy of the book, then browse, dip in and out, and share the book with friends and colleagues, because *99 Variations on a Proof* doesn't necessarily require a front-to-back read, but does provide a catalyst for discussions about the nature of proof and

[1] Raymond Queneau, *Exercises in Style*, 1947. Translated by Barbara Wright. London: John Calder, 2008.
[2] Gerolamo Cardano, *Ars Magna*, 1545.
[3] Oliver Bynre, *The First Six Books of The Elements of Euclid*, 1847. Edited by Werner Oechslin. Cologne: Taschen Books, 2010.

mathematics. Indeed, as part of a recent reading group of Lakatos's *Proofs and Refutations* we discussed the difference between formal and informal proofs, and a colleague of mine sent out several examples from *99 Variations* to illustrate the ways in which mathematics can be said to be formal.

A necessary result of the book's format is that it prioritises breadth over depth, meaning that the author gives us perspectives on many aspects of mathematical practice, but at times left me wanting more details and explanations. Of course, this is unavoidable, and the book is well-referenced with endnotes and a detailed bibliography to point the reader onwards to other sources.

To give a mild criticism, I do think there was a missed opportunity to mention some of the big philosophical questions that the proof variations point us to: What is a proof? What makes two proofs distinct? What does it take for a proof to be rigorous? How does proof presentation affect our understanding of the theorem? For example, the book presents ninety-nine "variations" on a proof, but really some are variations of the same proof, some are entirely distinct proofs, and some not proofs at all. Where to draw the line between these is an open question. The reason this could've been a nice chance to explore these questions is that the contrast between the variations naturally exposes the different values, virtues and vices these different proofs have, as well as the different insights and ideas different presentations provide. Ording himself tells us that "[m]athematical proofs often emerge as a sequence of individually inconsequential transformations governed by external constraints." (p. 22) The book provides a clear demonstration of this thesis that variation leads to mathematical discovery, and that small changes can lead to dramatic shifts of understanding.

Nevertheless, this missed opportunity is hardly a complaint at all compared to how fantastic *99 Variations on a Proof* is. I was constantly surprised by how much there was for me to still learn about such a relatively simple piece of mathematics, and how different representations clearly and succinctly brought out new ideas. While Queneau's *Exercises in Style* is rather silly and sets out to parody the styles it uses, Ording's *99 Variations* is a tribute to the past and present of mathematics in all its forms, and perfectly moderates the snatches of silliness with genuine insights and observations of mathematics as a broad and varied discipline.

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