GREGORY'S PILLAR

People, Places, Practices

BSHM - CSHPM/SCHPM Conference

Coordinated through the
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Abstract booklet

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Acknowledgements

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Isobel Falconer (lead organiser)
St Andrews,
July 2021

Introduction

Coming soon

Social Activities

Social Brunch

For an hour before the first plenary, each day

Brigitte Stenhouse: *Pub Quiz*

Monday after last plenary

Mark McCartney, Sarah Hart, Snezana Lawrence and Brendan Larvor: *The Unbelievable Truth*

History of maths version of a popular BBC Radio 4 panel show
Tuesday after last plenary

Calum Naughton, Kate Hindle & Others: *Board Games Social*

Wednesday after last plenary

Kenneth Falconer: *Another Quiz*

This one will be in random teams
Thursday after last plenary

Live Activities

(all in zoom room 3)

Moira Chas: *Play or poetry workshop*

Isobel Falconer: *Special Collections "Show and Tell"*

St Andrew's University holds outstanding collections of historically important mathematics books and manuscripts. Based around images, this talk will give an overview of some of the collections and how the University acquired them. We are grateful to Pilar Gil (Cataloguing and Documentation Officer) and Rachel Hart (Senior Archivist), University of St Andrews Special Collections, for sorting out and supplying the images. Special Collections website: https://www.st-andrews.ac.uk/library/special-collections/

Deborah Kent, Ritwik Anand, Alisz Reed and Mohak Misra: *Mathematically Curious St Andrews II*

Students from the University of St Andrews will speak about a virtual walking tour they created that focuses on the history of mathematics around the town of St Andrews. An overview of the project will be presented and access to the tour map will be shared. Some of the sites will be discussed and the process behind the research explained. The tour will feature different sites from Mathematically Curious St Andrews I on Monday.

Sophie Lenihan: *Exhibit: Digital Storytelling*

Find out how to use St Andrews' innovative new Exhibit tool, which enables anyone to access our digitised objects and to create interactive presentations with material from our collections. Exhibit was initiated during the Covid-19 pandemic in response to the challenge of providing an engaging and interactive experience using the museums and special collections. It addresses the sensory and tactile encounters students would have with this original material. Exhibit was developed by University of St Andrews and Mnemoscene using The Universal Viewer, an open source IIIF viewer used internationally by galleries, libraries, archives and museums, with support from the Esmée Fairbairn Collections Fund. https://exhibit.so/

Open Virtual Worlds (Bess Rhodes & Alan Miller): *Tour of medieval and early modern St Andrews*

The tour will include re-creations of St Salvator's Quad, ….. The Open Virtual Worlds group at St Andrew's University is a multidisciplinary group focussed on the application of digital technologies for the preservation and promotion of both cultural and natural heritage. We work with immersive and mobile technologies to discover how to create engaging interactive experiences. https://www.openvirtualworlds.org/

Gerry O'Reilly, Jessica McClure, Bronte Stones and Yansong Li: *Mathematically Curious St Andrews I*

Students from the University of St Andrews will speak about a virtual walking tour they created that focuses on the history of mathematics around the town of St Andrews. An overview of the project will be presented and access to the tour map will be shared. Some of the sites will be discussed and the process behind the research explained. The tour will feature different sites from Mathematically Curious St Andrews II on Thursday.

Plenary Talks

All in zoom room 3

Evelyn Barbin: *Introducing an historical perspective in mathematical teaching: meaning and practices*

Since the end of the 1970s, the French IREMs (Institutes for Research on Mathematics Education) have promoted the introduction of "a historical perspective into the teaching of mathematics". In one, preferable, approach, this expression means the integration of all the historical and epistemological knowledge of a teacher in their teaching. It brings into play the profound modifications that this knowledge can bring to their conception of mathematics and to their classroom practices. We will examine these points in opposition to a teaching of mathematics containing "historical moments" specially arranged for teachers who have never had the opportunity to study and work the history of mathematics.

Raymond Flood and Robin Wilson: *The BSHM - The first fifty years*

The British Society for the History of Mathematics (BSHM), founded in 1971 and the oldest society for the history of mathematics, is celebrating its half-century. This talk, which is dedicated to the memory of Peter Neumann, describes how and why the Society was founded, and what it has achieved over its first fifty years.

Valeria Giardino and Frederic Patras: *Proving with graphs: the mathematician's toolkit*

An increasing number of studies in philosophy of mathematics have been recently devoted to the analysis of segments of the contemporary practice. An important element of the practice of mathematics is proof, and mathematical proofs are often supplemented by diagrams and by a specific terminology. As Azzouni [1] has suggested,
there is no reason to make every step in rigorous mathematical proofs entirely explicit; rather, it should be recognized that proofs are based on schematic rules that apply to classes of arguments; he frames this process in terms of a kind of "know-how" and tacit knowledge; mathematical intuition so defined involves "inferential packages" that are applied to the proof as 'black box units'. Differently from Azzouni, who focused on Euclidean geometry, Carter [2] discussed an example of diagrammatic reasoning that does not take place in geometry and shows that some of the formal definitions in the final paper arise from original proofs where diagrams were massively employed, irrespectively of the fact that they were dropped out in the final publication. The objective of the present paper will be to introduce and discuss a case study of an article published by A. Connes and D. Kreimer in 2002 [3] where diagrams–Feynman diagrams in particular–are shown and used in part of the proof and accepted as displaying valid arguments. The hope is that this will pave the way to a wider discussion of the role of several kinds of diagrams across different practices.

References:

[1] J. Azzouni (2005), Is there still a Sense in which Mathematics can have Foundations? In: G. Sica (ed.) *Essays on the Foundations of Mathematics and Logic*, Polimetrica International Scientific Publisher Monza/Italy, 9-47.

[2] J. Carter (2010), Diagrams and Proofs in Analysis. *International Studies in the Philosophy of Science*, 24:1, 1-14.

[3] A. Connes and D. Kreimer (2002), Insertion and Elimination: the Doubly Infinite Lie Algebra of Feynman Graphs. *Annales Henri Poincaré*, 3, 411- 433.

Brendan Larvor: *Mathematical Memes in the Age of Reason: the limits of understanding and the understanding of limits*

David Hume devoted a long section of his Treatise of Human Nature to an attempt to refute the indivisibility of space and time. In the later Enquiry Concerning Human Understanding, he ridiculed the doctrine of infinitesimals and the paradox of the angle of contact between a circle and a tangent. Following up Hume's mathematical references reveals the role that precisely these mathematical examples (the indivisibility of space and the angle of contact) played in the work of philosophers who (like Hume) were not otherwise interested in mathematics, and who used them to argue for either fideist or sceptical conclusions. That is to say, this handful of paradoxes were taken to mark the limit of rational mathematical enquiry, beyond which human thought should either fall silent or surrender to religious faith.

This argument occurs, notably, in Malezieu's *Éléments de Géometrie*, to which Hume refers indirectly in the Treatise. This book was an elementary textbook suitable for aristocratic youth, and therefore far from the discourse of the mathematical elite. The fact that it was the same pair of examples turning up in extra-mathematical books (such as the Port Royal Logic) or elementary mathematical writing (such as Malezieu's *Elements*), without ever including obvious alternative candidates such as Torricelli's horn of plenty, indicates that they constituted a stable unit of discourse that was reproduced without further reference to mathematical literature or expertise—a meme.

Following Hume's mathematical sources shows us something about the role and significance of mathematics in the wider intellectual culture of his time. A small number of isolated and fossilized puzzles became emblematic of mathematics as both rational authority and inaccessible mystery.

Colm Mulcahy: *The Scottish Irish mathematical trail*

John O'Connor and Edmund Robertson: *A history of the development of MacTutor*

The Web version of the MacTutor archive started as an adjunct to our teaching software and much of its early design was strongly influenced by that.

As we grew from our small beginnings and other sources on the history of mathematics became available, we have changed both the way we present our material and our approach to writing biographies.

A recent redesign of our site has brought it up to date, but our origins are still detectible in the modern version.

In this talk we describe how our approach to developing MacTutor has changed over the last 33 years.

Karen Parshall: *People in New Places: The American Mathematical Research Community and the First Wave of Mathematical Émigrés, 1933-1938*

As is well-known, in April 1933, Nazi policies compelled the expulsion of so-called non-Aryans from German universities. That act set in motion a flight of Jewish and other mathematicians—first from Germany and then from other European countries—as Hitler gained power and began the rampage across Europe that would result in the outbreak in 1939 of the Second World War.

Those in the first wave, however, arrived when colleges and universities were still in the throes of the Depression. They were perceived by some as depriving deserving American mathematicians of positions. It also seemed that they were being provided with more research-oriented—as opposed to time-intensive teaching—positions because of their perhaps imperfect command of English and their lack of familiarity with the American undergraduate. The majority of them, moreover, were Jewish at a time when anti-Semitism was far from unknown in academic and other American social circles.

Could they be absorbed and, if so, in ways that strengthened rather than undermined the American mathematical community? Activists like Oswald Veblen and Roland Richardson hoped that they could but recognized equally that success would hinge on the ability of the transplanted Europeans and the home-grown Americans mutually to adapt. This talk will explore the dynamics of that process.

Robin Wilson: *The diaries of Thomas Archer Hirst – Mathematician Xtravagant*

Although little remembered nowadays, Thomas Archer Hirst (1830-92) was a major figure in British Science in the Victorian era.

Elected a Fellow of the Royal Society at age 31, he became President of the London Mathematical Society, Professor of both Physics and Mathematics at University College London, General Secretary of the British Association, a founder-member (with Tyndall and Huxley) of the influential X-club, and the first Director of Studies at the Royal Naval College at Greenwich. He was also an inveterate traveller who met Gauss, carried out physics experiments with Weber, and knew large numbers of Continental mathematicians, such as Dirichlet, Steiner, Liouville, Chasles, Chebyshev and Klein.

But despite all these achievements, he would have become a forgotten figure had it not been for his habit of writing a journal which tells us much about scientific life in the Victorian era, with vivid descriptions of Faraday, Darwin, Boole, Maxwell, Cayley, Sylvester, De Morgan, and many others. Covering 45 years and extending to over 2 million words, his extensive diaries chronicle with great clarity the scientific and social circles in which he moved – both in England and Europe.

Symposia Abstracts

Verity Allan, Mary Monro, Ursula Martin: *Women in British Computing in the 20th Century*

Overview

The contribution of women to computing in the 20th century is well hidden and seldom recognised. Unlike men, their names seldom appear on scientific papers; frequently they are not even acknowledged. Tracking their identities, achievements, and working conditions, is a treasure hunt through archives, photographs and personnel records. This session explores the practices of such women from the inter-war period when “computers were human”, through to the 1990s. It investigates them as innovative users and as entrepreneurial suppliers of computing, essential in meeting the rising demand for complex data analysis in university, government and commercial contexts.

Mary Monro: *Dora Metcalf, entrepreneur and mathematician*

Dora Metcalf was a mathematician who could see the potential of mathematics for business and governments: she was first and foremost an entrepreneur. Emerging technology - first, comptometers and, later, punch card electromechanical computers - brought her ideas to life and made them achievable. Dora gained an external degree in mathematics from London University in 1911. She established a business selling comptometers in Belfast in 1916. She subsequently joined forces with her cousin, Everard Greene, a founder of the British Tabulating Machine Company (BTM) to win the contract for the 1926 Northern Ireland census, and later became a divisional director of BTM. Dora pioneered the provision of computation services in the 1920s and ‘30s across the British Empire, including Pope Pius XI’s charities. In the Second World War BTM built over 200 ‘bombe’ machines for the codebreakers at Bletchley Park, crucial to cracking the Enigma code. Post-war she built up BTM further, selling comptometers, computers and computing services. She was far-sighted in understanding these new tools, selling or renting ‘proto-computers’ to big business and providing computation services to small businesses.

Verity Allan: *Women “computors” in radio astronomy at Cambridge*

In the period after the Second World War, the University of Cambridge enjoyed a period of great scientific success. Some of this science relied heavily on the Fourier Transform, in particular, x-ray crystallography and radio astronomy. People (primarily women) were hired to do the Fourier Transform calculations, initially by hand.

At the same time, Cambridge was part of the computer revolution, and many of these women moved to (or later were hired for) programming the innovative computers developed at the University's Mathematical Laboratory.

The contribution of all computing staff was essential; for radio astronomy, computers enabled larger and more complex telescopes to be built. Computing thus underpinned the Nobel Prize awarded to Ryle and Hewish in the 1970s for their innovative telescope design. Conversely, the demands placed on computers by the science required the development and employment of new and complex computing techniques, as well as a deep understanding of the scientific and mathematical underpinnings of the problem.

My work focusses on the women "computors" working in radio astronomy at the Cavendish Laboratory, and the techniques we use to uncover their contribution.

Ursula Martin: *Early Women in computing at Oxford*

We are investigating women who were involved in the University of Oxford's pioneering work in computing, either as staff, users or students, including a series of oral history interviews by Georgina Ferry for the Bodleian.

Dorothy Crowfoot Hodgkin, who won the Nobel Prize in 1964 for her work on the structure of insulin, was an early adopter of computing, heading the team who chose Oxford's first digital computer in 1959. The four founders of the Oxford-based numerical software company NAG, celebrating its 50th anniversary in 2020, included Linda Hayes, Shirley Lill (now Carter), and the late Joan Walsh. Women were leaders in Oxford's pioneering work in Digital Humanities, with Susan Hockey developing the first concordance programs for Arabic and Turkish.

These stories reveal the complex ecology that developed around the teaching, research and practice of British academic computing between the 1950s and the 1990s, in which women played essential but often unacknowledged roles, often moving on to significant roles in academia and industry.

June Barrow Green, Reinhard Siegmund Schultze and Alison Maidment: *British and German applied mathematics around 1900 – some commonalities and some differences as exemplified by Henrici, Whittaker, and von Mises*

Overview

The symposium aims to look at innovations around 1900 in applied mathematics within a cross-cultural comparison between Germany and Great Britain.

June Barrow-Green: *The* mathematical *models and machines of Olaus Henrici*

The role of mathematical models and of new calculational devices will be stressed in the case of the English engineer and mathematician with German origin, Olaus Henrici, who acted as an envoy and cultural ambassador between the two countries. Henrici, who was educated in Karlsrhuhe under Redetenbacher and Clebsch, and later wrote a thesis on algebraic geometry with Hesse in Heildelberg, made his career in England, moving there in 1865. He began his working life as an engineer but moved into the teaching of mathematics at University College London, where he was a proponent of projective geometry. And it was during this period that he constructed several models of geometric surfaces. After his move in 1884 to the Central Institution in South Kensington, where he was teaching engineers, he opened a laboratory of mechanics, a novel institution in its day. This was also the setting for the production of his harmonic analysers, the machines for which he is now best known. In the wider scientific sphere, he contributed models to the 1876 *South Kensington Museum Special Loan Exhibition of Scientific Apparatus*, and he played a significant role in the 1893 *Mathematical Models, Apparatus and Instruments Exhibition* in Munich, organised by the Deutsche Mathematiker-Vereinigung. For the latter he not only lent geometric models and his harmonic analyser, but also led the organisation of the loan of British exhibits (the biggest contribution from outside Germany), and wrote extensively for the catalogue.

Alison Maidment: *ET Whittaker's mathematical laboratory*

In the case of the Scottish mathematician of English origin, Edmund Taylor Whittaker, the rising role of mathematical laboratories and of numerical analysis comes into the picture, a common feature throughout Europe at the time. In 1912 Whittaker, a Cambridge graduate, moved from Dunsink Observatory, where he had been Royal Astronomer of Ireland, to Edinburgh to take up the chair of mathematics. The following year, motivated by his experiences in Ireland and the example of Runge in Germany, as well as his association with several actuaries in Edinburgh, he opened his mathematical laboratory, the first of its kind in Britain, and the setting for the "the practical instruction in numerical, graphical, and mechanical calculation and analysis". In 1913, the laboratory played host to a 5-day series of lectures by Whittaker as part of the inaugural Edinburgh Mathematical Colloquium. The laboratory was the stimulus for several textbooks on various aspects of numerical analysis, most notably Whittaker & Robinson's *The Calculus of Observations* (1924). It also had a direct influence on the establishment of other mathematical laboratories and publications both in Britain and in the United States.

Reinhard Siegmund-Schultze: *Richard von Mises, Hilda Geiringer and their "Praktikum" for applied mathematics at Berlin University during the 1920s*

On the German side, the later developments are exemplified by the mathematician and engineer of Austrian origin Richard von Mises, who founded both a famous school and the internationally leading journal in Berlin during the 1920s, and who considered British engineers such as Reynolds and Rankine as role models. Between 1907 and 1911 von Mises collaborated with Felix Klein in Göttingen, the noted reformer of German applied mathematics, on the "Encyclopedia of Mathematics and Bordering Subjects." Both Klein and von Mises were very internationally minded. In 1920 von Mises founded ZAMM, the Journal of Applied Mathematics and Mechanics which Klein welcomed as "the first institutional frame for engineers and mathematicians to find common ground" and at the same time a role models for other journals of applied mathematics to come. Von Mises' "Praktikum" at Berlin University – somewhat comparable to Whittaker's laboratory – became famous as a place for training the new generation of applied mathematicians. Among them was Hilda Geiringer, von Mises' assistant and future wife, who became an international specialist in plasticity theory. Another even more famous student of von Mises was Lothar Collatz, the pioneer of modern numerical analysis, who credited von Mises' lectures around 1930 for inspiring him to take this path.

Eduardo Dorrego Lopez, Elias Fuentes Guillen and Jose Manuel Ferreiros Dominguez: *Real Numbers in Transition. Aspects of the 18th and 19th centuries*

Overview

It is usually accepted that the notion of real numbers was more or less there prior to the publication of detailed theories (often called constructions) in the 1870s. However, a closer look indicates that the extent of the domain of real numbers was far from well determined, or well understood, in the 18th and early 19th centuries. Up until 1800 it was still common to understand by irrational numbers only the radicals such as $\sqrt{3}$ or $\sqrt{1-\sqrt{5}}$ but gradually there was a transition from the conception of decimal fractions (or continued fractions, etc.) as a tool for approaching irrationals, to their conception as mathematical objects. Meanwhile, the preeminence of analytic methods introduced in the 17th century made possible some proofs of the irrationality of transcendental numbers during the 18th century. At the turn of the 19th century the need for a clear conceptual characterization of the continuity of the real-number domain was not yet been envisioned and indeed crucial ingredients of a mature theory of real numbers were still lacking, as evidenced by the sporadic introduction of irrationals. But this is not to deny some groundbreaking features contained in the works of mathematicians of the first half of that century.

The aim of this symposium will therefore be to throw light on that transitional period, before the 1870s, by examining the relevant contributions due to key authors of the German-speaking area, namely Lambert, Bolzano, M. Ohm and Grassmann. In doing so, and partly by paying attention to their biographies, we intend to address the ways in which those mathematicians and some of their contemporaries worked during that period.

Eduardo Dorrego Lopez: *The 18th century as the germ of an in-depth understanding of irrational numbers.*

Elias Fuentes Guillen: *The tensions underlying Bolzano' Groössenlehre.*

Jose Manuel Ferreiros Dominguez: *The transitional contributions of M. Ohm and Grassmann*

David Dunning, Jenne O'Brien, Henning Heller and Tabea Rohr: *Informal Formalization*

Overview

The story of mathematics around 1900 is largely a story of formalization. A central narrative in the history of modern mathematics emphasizes the discipline's turn away from physical applications toward a formalized world of spaces, structures, and sets. Indeed historical investigation has often adopted precisely the formal outlook it should seek to explain, taking the development of formal concepts as co-extensive with the history of mathematics. Recently the historiography of mathematics has taken a welcome turn away from exclusive internalism. But the history of formalization *itself* continues to present a challenge for other approaches insofar as the very objects of historical inquiry, the formal concepts and axioms whose histories we seek to understand from multiple perspectives, suggest the teleology of their own formal definitions at the outset.

This symposium asks whether formalization—the human activity of formalizing one's mathematics—is always formal work. We probe the informal cultural practices that sustain formal mathematics, suggesting that the formal and the informal are two sides of the same coin. How, in specific historical instances, did the modern formalization of pure mathematics take place? How did informal activities contribute to the production of something that could then be widely recognized as formalized? How might the actors' category "formal" exert unseen influence on our efforts to write history (and philosophy) in the present?

Jenne O'Brien: *Reading and Re-forming: the Creation of Bernhard Riemann's Habilitation Lecture*

In his Habilitation lecture "On the Hypotheses which Underlie Geometry," Bernhard Riemann defined the manifold concept, an abstraction that distanced geometry from its ties to physical space and enabled more formal treatments of space. In this talk, I focus on Riemann's construction of the lecture in the context of his reading, and more specifically his appropriating, of other scholarship. I show that Riemann did not so much import parts of texts directly into his lecture, but rather transformed these textual fragments through "abstraction" and "generalization" (his words) in surprising ways to create novel scholarship. Not only that, Riemann looked to the works of other scholars for the methods of abstraction and generalization he used in appropriating; in other words, he appropriated methods of appropriation. In short, if formalization has historically served to create consensus by confining possible ways of reading, this paper examines a way that one mathematician read and engaged with scholarship prior to formalization.

Henning Heller: *Formalization in 19th century group theory and the emergence of quotients groups*

Within the history of 19th century group theory, the development of the concept of a *quotient group* has so far received very poor attention. Although implicit uses of quotient groups can be traced back to Jordan (1870, 1873) and even Galois (1832), it is generally agreed that the concept was first explicitly acknowledged as late as 1889, when Hölder provided a definition based on an abstract conception of groups. This occasion led to the now-dominant narrative according to which the concept of a quotient group could only be fully grasped after group theory was sufficiently formalized (Cf. Wussing 1984, Schlimm 2008). However, some so-far overlooked lecture notes of Felix Klein's algebra course in the summer of 1886 might force us to renounce this narrative. Especially Klein's repeated use of the *Correspondence Theorem* for groups unveils a sophisticated understanding of quotient groups independent from any attempts of formalization. More than that, I want to argue that the parallel process of formalization was in part motivated by the need to capture concepts like quotient groups and the related Correspondence Theorem (as well as an increased wealth of examples of specific groups and theorems), not a mere "by-product". This idea reverses the commonly accepted dependence relation between formalization and the formalized.

Tabea Rohr: *Hilbert’s Axiomatization in Context*

In this talk Hilbert’s formalization of Geometry in his *Grundlagen* will be set into the context of the 19th century dispute between analytic and synthetic geometers. Their arguments for the one or the other approach to geometry were not purely geometrical, but they attributed different epistemic values to them. Hilbert gave in the early 1890’s lectures on this different geometrical methods and put his own axiomatic approach into that context, arguing that it combined the epistemic advantages of both. It will be shown, how Hilbert’s choice of axioms and models was shaped by this background.

David E. Dunning: *Formal Foolery: Symbolic Notations and Play in the History of Abstract Mathematics*

A central task in any project of formalization is to demarcate what belongs and what does not belong to the system. I propose to understand formalization as a human cultural practice by exploring the ways users engage in activities that are obviously external to the project—uses that simply seem to offer some sort of fun. I will consider three case studies. Gottlob Frege, who faced a perennial struggle to convince others to use his Begriffsschrift notation, found his greatest pedagogical success in students who used it send casual messages about making social plans. Alfred North Whitehead and Bertrand Russell sprinkled jokes throughout their nearly unreadable *Principia Mathematica*, suggesting that the monumental ambitions of their logicism did not exhaust the cultural meaning they saw in the project. In Interwar Warsaw, a hotbed of formal logical innovation, researchers practiced what one postwar logician described as the “peculiar intellectual sport” of seeking typographically short axiom systems. Together these studies demonstrate the multiple functions of play with respect to formal systems, with seemingly frivolous activities producing the sociability, comic relief, and friendly competition that strengthened social communities of formalizing researchers.

Meredith Houlton, Philippe Schmid, M. Pilar Gil and Ella Duréault : *Mathematicians, Astronomers and Innovators at the University of St Andrews in the Early Modern Period*

Overview

Innovative science and maths in Scotland in the late seventeenth century has been little explored compared to the later Scottish Enlightenment. During the late seventeenth century there was a flourishing of intellectual thought and innovation sometimes overlooked. These talks will cover topics including the student experience and the progress that occurred and will formulate a picture of the people and their activities in the locale of St Andrews during the early modern period.

Ella Duréault & Pilar Gil: *Planning the observatory of St Andrews: The role of scientific instruments in the 17th century*

Examining the networks of communication and exchange casts a new light on the way James Gregory gathered astronomical instruments for the Observatory.
Indeed, the mathematician played a pioneering role in the introduction of proper scientific instrument-making at the university. Before arriving in St Andrews, he had already written *Optica Promota* (1663), where he was the first to describe the principle of the reflecting telescope.

His correspondence with great English astronomers such as John Collins and John Flamsteed, the Royal Astronomer, reveals that Gregory was part of a network of mathematicians and instrument-makers which largely contributed to helping him to furnish the Observatory.

These observations allow us to consider that the foundation of the Observatory was not an isolated initiative. Studying the process of instruments' acquisition by Gregory is an interesting touchstone to undertake the relationship between theory and practice in astronomy in the second part of the seventeenth century.

Meredith Houlton: *The Elements of Geometry*, by William Sanders

William Sanders was appointed to the Chair of Philosophy at St Andrews in 1672. When James Gregory left St Andrews for the University of Edinburgh, Sanders became Regius Chair of Mathematics in 1674, and in 1686 Sanders published *Elementa Geometriae*. The book was written in Latin, and there is no extant translation.

Examination of Sanders' text indicates how he may have used it with students and what his intentions may have been writing and publishing it. This research explores the mathematics of Sanders' book and demonstrates that Sanders combined a classical Euclidean approach with Early Modern concepts by referencing Early Modern mathematicians.

Other Early Modern mathematics texts were consulted. Insights were provided into William Sanders' persona as an educator, mathematician, and Regius Chair and also into the nature of mathematical education and texts in Scotland in the late 17th century.

Philippe Schmid: *Student Note-Taking and Mathematical Diagrams at the University of St Andrews*

The practice of student note-taking at Scottish universities has recently been studied by Matthew Eddy, who focused on notebooks written after the Jacobite Rebellion in 1745. Eddy casts student notebooks as 'papertools' that use a series of graphic or scribal practices to become 'an interactive platform of information management for students and professors'. He differentiates three genres of Scottish student notebooks: the *manuscript textbook* of the seventeenth century; the *commonplace book*; and the *lecture notebook* of the eighteenth century, on which his article was based. This paper approaches the first category of the manuscript textbook of students produced *before* the Scottish Enlightenment. It studies the practices of student note-taking at the University of St Andrews in the late seventeenth century, following the career of the student Colin Campbell, who matriculated at St Leonard's College in 1676 and graduated in 1679. Two series of lecture notes survive, which were written by Campbell during his lectures with Alexander Cockburn, who became regent at the college in 1676, and who was a partner of the mathematicians James Gregory and William Sanders. The first lecture covers the mandatory philosophy course, including logic, metaphysics and physics; the second lecture deals with astronomy. Building on the research of Ann Blair, Matthew Eddy and Paul Nelles, I will present the stages involved in the scribal production of a manuscript textbook in Restoration Scotland. In a second step, I will discuss the functions of the mathematical diagrams in these notes. I will argue that manuscript notebooks did not only follow the typographical models of print textbooks, as Eddy has shown, but also imitated the new scientific journals of the 1670s.

Deborah Kant, Benjamin Wilck and Yacin Hamami: *Mathematics and the dialogue: a symposium*

Overview

The aim of the symposium is to survey and evaluate various forms of dialogical argument in the practice and the philosophy of mathematics. We consider the dialogue as a tool to learn about mathematical practices, and as a philosophical concept to model mathematical practices. Mathematicians and philosophers of mathematics alike make use of dialogical forms of argument in contexts of both scientific discovery and scientific justification, especially pertaining to foundational issues such as the justification of axioms. Each of the three talks assembled in the symposium address genuinely dialogical strategies of mathematical reasoning and inquiry from a different point of view. While the first talk reconstructs the way in which Aristotle appeals to the Socratic dialogue to examine and refute putative mathematical definitions, the second talk presents a method of interviewing contemporary mathematicians about their respective meta-mathematical background assumptions. The third talk discusses dialogical aspects of rigor judgments in mathematical practice.

Benjamin Wilck argues that Aristotle's account of dialectic in the *Topics*, which is a codification of the Socratic dialogue, serves as a dialogical method to examine putative scientific principles. The dialectical game involves two rival parties: the questioner and the respondent. The respondent's task is to put forward a proposition (for instance, a mathematical definition) and to defend it against the questioner's attack. The questioner's task is to examine the respondent's proposition on the basis of the respondent's concessions in dialectical debate. In particular, the questioner seeks to refute the respondent by checking whether the respondent's proffered proposition is inconsistent with the respondent's concessions. Aristotle expressly takes dialectic to be designed as a testing procedure for putative scientific principles such as mathematical definitions and axioms. Consequently, putative scientific principles are dialectically examined on the basis of the respondent's concessions alone. However, why should a scientist care about dialectical testing of her proffered definitions or axioms, given that science aims at truth, whereas the respondent's concessions may be false? The paper argues that, surprisingly, dialectical testing is effective precisely because dialectical arguments rest upon premises that need not be true, but need only be granted by the respective respondent

Deborah Kant employs a form of methodological dialogue in her interview study with professional set theorists about their work and views pertaining to the independence phenomenon. These interviews exemplify a specific form of dialogue between a philosopher and a mathematician. The questions are formulated according to methodological criteria from the Social Sciences. These interview questions thus have a genuinely argumentative function, but they are different from the kind of philosophical question that we find in the Socratic dialogues. Although there may be appropriate methods other than the qualitative interview to study the set-theoretic community's research practices and meta-views concerning set-theoretic research such as studying mathematical literature, or mathematical communication between the practitioners, or one could do surveys instead of interviews. However, there are two major advantages in performing a dialogue between the investigating philosopher and the mathematician who is part of the community to be investigated. Firstly, the philosopher is likely to get more information in an open, qualitative interview study and, secondly, most other methods presume that philosophers have a detailed, descriptive account of set-theoretic practices that can be tested, while the goal of an open qualitative interview study is rather to construe such an account.

Yacin Hamami investigates the specific type of dialogue that arises when the rigor or correctness of a mathematical proof is evaluated in mathematical practice. When a mathematician, or a group thereof, claims to have provided a rigorous proof of a mathematical theorem, the proof is then examined within the relevant mathematical community. In particular, members of the community can then challenge different parts of the proof by pointing out gaps in the reasoning or by exhibiting counter-examples to some of the inferences. These challenges call for responses from the author(s) who can respond by providing further details, by adjusting the formulation of the theorem, and if no response is available, by withdrawing the claim that the proof provided is rigorous. This dialogue can take place in various contexts such as discussions in workshops and conferences, during the peer-review process, or through personal communications. In this talk, Y. Hamami will propose to construe this dialogue as a game in which a defender aims to defend her claim that a certain mathematical proof P is rigorous against a challenger whose aim is to challenge this claim—he will refer to this game as the rigor game associated to P. In practice, both the defender and the challenger can be either a single mathematician or a group thereof. He will then specify the structure of rigor games, and he will argue that there is an intimate connection between correctly evaluating the rigor of a mathematical proof P in practice and possessing a winning strategy against the challenger in the rigor game associated to P.

Jemma Lorenat, Della Dumbaugh, Laura Turner, Sloan Despeaux and Marjorie Senechal: *Invisible power: journal editors and the publication of mathematics*

Overview

Recent historical scholarship has highlighted the mathematical journal as a major vector of dissemination and circulation of mathematics from the early nineteenth century to the present. In this session the emphasis will shift toward the locus of power embodied in a journal's editor.

Though it is well known that editors had the authority to accept or reject individual publications, the collective process of publishing mathematics could extend from the choice of subject-area to the final word choice, textual emphasis, and use of footnotes. Besides the written content, the journal editor also navigated considerations that were practical --- including financial concerns, readership volume, and what kinds of mathematical symbols were fit to print --- and even political, particularly as nations were built and during periods of war.

The four speakers draw on research, educational, and popular mathematical publications from the turn of the twentieth century: *Acta Mathematica*, *The Annals of Mathematics*, *Educational Times*, and *The Monist*. Notably, given the shifting barriers between these three types of exposition, it would be possible for a mathematician to have published in all four journals. These journals were published in Europe and the United States, but also included contributions from Asia and the Americas. In this international arena, the editor could nurture and promote inclusion or establish barriers.

The variety of historical editorial models explored in this session all exist to this day alongside the tenure-sanctioned double-blind peer review standard. To reflect on contemporary echos, the session will conclude with commentary from Marjorie Senechal, who has served as the editor for *The Mathematical Intelligencer* --- a journal of mathematical culture --- for most of the twenty-first century. Senechal draws on her personal experience and expertise as a historian of mathematics.

Della Dumbaugh: *Solomon Lefschetz as editor of the* Annals of Mathematics

In late 1907, at the age of 23, the engineer Solomon Lefschetz lost his hands and forearms in a transformer accident. This unfortunate situation prompted Lefschetz to reorient his life towards a career in mathematics where he ultimately played a critical role in the American mathematical community in the twentieth century. He contributed significantly to algebraic topology, its applications to algebraic geometry, and the theory of non-linear ordinary differential equations. He demonstrated his leadership as a faculty member at the University of Princeton and as President of the American Mathematical Society, among other influential positions. Lefschetz also served as the main editor for the *Annals of Mathematics* from 1928 to 1958, an important period for the journal. During this time, the *Annals* became an increasingly well-known and respected journal. Its rise, in turn, stimulated American mathematics. This talk introduces Lefschetz and explores his role as editor of the *Annals*, including the papers that were published in the journal, the editorial boards, and the authors of the more than 1850 articles that appeared during his editorship.

Laura Turner: *Gösta Mitag-Leffler as editor of* Acta Mathematica

In 1882 Swedish mathematician Gösta Mittag-Leffler (1846-1927) founded the journal *Acta Mathematica* and served as its editor-in-chief, supported by a board of eminent Scandinavian mathematicians, until his death. Intended to serve mathematicians of all countries, and also to showcase Scandinavian research contributions to foreign scholars, the creation of *Acta* reflected a growing interest in the internationalization of mathematics during a period marked by the rise of nation-states and nationalism. So, too, did Mittag-Leffler's narratives surrounding the success of the journal, and his efforts to ensure its survival as he navigated both practical considerations and critical geopolitical events.

In this talk, we will explore some of the ways in which Mittag-Leffler's role as editor afforded him the power not only to determine which individuals, subjects, nations, and languages would feature in the pages of *Acta Mathematica*, but to make such decisions so as to assert the position of his nascent Stockholm research school, Sweden, and Scandinavia, more broadly, on the international stage.

Sloan Despeaux: *William John Clarke Miller as editor of the* Educational Times

William John Clarke Miller (1832-1903), an active contributor to mathematical column of the *Educational Times,* took over the editorship of this column in 1862. Over the next three decades, Miller used his position as editor to change the target audience, contributors, and space devoted to this column.

Jemma Lorenat: *Paul Carus as editor of* The Monist

The Monist began publication in 1890 as a journal ``devoted to the philosophy of science'' and dedicated to bringing European (particularly German) texts to American readers. From the first volume, *The Monist* regularly featured mathematical content. Many of the local contributors considered themselves knowledgeable amateurs, and published alongside turn-of-the-century greats such as Poincaré, Hilbert, and Veblen. The mathematical content was carefully curated by the German émigré Paul Carus with help from a handpicked behind-the-scenes team of American mathematicians. Editorial correspondence from this period reveals regional tensions and racial biases that determined a popular face of mathematics that varied from recreations to logical foundations.

Anuj Misra and Hassan Ameni: *Linguistic hospitality in Arabic, Persian and Sanskrit mathematical works*

Overview

In the modern day, mathematics is almost universally expressed in a formal language of symbols and notations. However, from a historical perspective, mathematical ideas were first expressed in different languages by practitioners from different cultures. The vocabulary with which mathematics was articulated in different societies, at different times, and by different people followed the historical, cultural, and scientific traditions prevalent in those times and places. As these traditions changed with the passage of time and movement of people, so too did the locus and language of mathematics.

The capacity of an ordinary language to appropriate, assimilate, or innovate new words gives its speakers the conceptual space to interact with foreign ideas. Mathematics, when expressed in natural languages, shared in this benefit of conceptual diversity. Mathematical works in Arabic and Persian from the high to late Middle Ages, as well as Sanskrit works from the Early Modern Period, attest to a *linguistic hospitality*—the ability to inhabit a different language space than one's own with an ambition to understand and engage with extraneous ideas—that extends across and beyond the geopolitical boundaries of India and the eastern Islamicate worlds. Studies on these texts not only enrich our knowledge of historical mathematics but they also allow us to examine the people, paths, and purposes that shaped that knowledge.

On the occasion of the joint meeting of the British and Canadian societies for the history and philosophy of mathematics at St Andrews in July 2020, we propose to bring together scholars working on mathematical texts in the three aforementioned languages. The conference theme of *People, Places, Practices* offers an apposite occasion to examine the potency of natural languages as carriers of mathematical thought across time and space.

Anuj Misra

Medieval Arabic and Persian scholars from the Abbasid, Ghaznavid, Ilkhanate, Timurid, and Ottoman dynasties were familiar with elements of Indian and Greek astronomy. These can be seen in several works authored by scholars like al-Khwārizmī (*Zīj al-Sindhind*, c.820 CE), al-Bīrūnī (*al-Qānūn al-Masʿūdi*, c.1036 CE), al-Tūsī (*al-Tadhkira fī ʿilm al-hay'a*, c. 1250 CE; *Zīj-i Īlkhānī*, 1272 CE), al-Kāshī (*Zīj-i Khāqānī*, c.1420), Ulugh Beg (*Zīj-i Sultānī*, 1438–1439 CE), and al-Qūshjī (*Risālah dar hay'ah*, c.1450–1460 CE) to name a few.

However, it is beginning from the late 14 th century and culminating in the 17 th century that we find Islamicate ideas of planetary tables (zīj), observational instruments, and Ptolemaic planetary models mentioned in the literature of Sanskrit astral sciences. These *foreign* ideas were vigorously debated in several Sanskrit canons authored during this period—a period that marks the cognisant and conscious engagement of Indian and Islamicate sciences in early modern India.

In this talk, I will discuss the technical vocabulary, literary tropes, and cultural devices with which one such Sanskrit treatise, the *Sarvasiddhāntarāja* (1638 CE) of Nityānanda, assimilates and articulates Islamicate (Ptolemaic) astronomy in Sanskrit.

Hassan Ameni

From the 10th century, Iranian scientists started composing some of their works in Persian instead of Arabic—the principal scientific language of the time. One of the earliest instances of Persian mathematical literature is the *Kitāb al-tafhīm li-awā'īl ṣināʻat al-tanjīm* `Book of Instruction in the Elements of the Art of Astrology' by Abū Rayḥān al-Bīrūnī (973–1048 CE) that introduced over a hundred technical terms of mathematics. Through the early parts of the second millennium CE, the corpus of technical literature in Persian began to grow, and in particular, the period between the 13th and 15th centuries saw a dramatic rise in the number and complexity of Persian scientific texts. Naṣīr al-Dīn al-Ṭūsī (1201–1274 CE) is one such remarkable scholar who wrote voluminously in Persian as well as in Arabic.

The presence of several polymath scholars in this period, proficient in both Arabic and Persian, makes the interaction between these two languages an essential feature in understanding the historical picture of the Islamicate science in early modern times. Arabic technical terms, through the legacy of being translated from Greek over centuries, were well established in the language of science discourse. The profusion and circulation of technical works in Persian brought another language into the parlance of Islamicate science of the period a language. In this talk, I explore the hospitality with which Persian works incorporated Arabic terminologies, weaving them into Persian literary styles and scientific lexicons.

Richard Oosterhoff, Alexander Corrigan and Lewis Ashman: *The Practices of Mathematical Antiquaries in Early Modern Britain*

Overview

This workshop considers people defined by practices that should matter more to historians of science and mathematics, because they were as mathematical as humanist. We suggest a category that joins the history of mathematics to the history of scholarship: the mathematical antiquary. Antiquarian practices are most often linked to humanities or perhaps natural history, but in fact many antiquaries had mathematical interests, and some of their most distinctive practices were fundamentally about managing numbers (chronology, astronomical events, histories of currencies and their exchanges, history of maths, proofs of historical problems...). The category of 'mathematical antiquaries' therefore resists the assumption that humanities and mathematics are opposed. It also gives a much longer history to the overlapping practices of digital archives and data management that are so often framed as distinctively modern.

Alexander Corrigan: *Mathematics of Polemics in Napier's* Plaine Discovery (1593)

In his *Plaine Discovery of the Whole Revelation of Saint John*, John Napier of Merchiston (1550–1614) developed a system for interpreting historical events as fulfilments of Biblical prophecy, and for predicting the future, which was unparalleled in its mathematical precision. Napier exceeded previous scholars like John Bale (1495–1563) in perceiving the past as overwhelmingly negative, being typified by the oppression of God's true church by that of the Antichrist. For Napier, historical enquiry had no intrinsic value as a sacred endeavour; his antiquarian interests were dictated by his polemical aims. His scheme aimed to convince his compatriots the world would soon end, and they must act immediately to expel all Catholics from the British Isles. This paper discusses the unusual aspects of that work. No previous commentary on the Book of Revelation in English had mathematical calculations as such an essential component.

Richard Oosterhoff: *Brian Twyne on the History of Mathematical Practice*

As the first Keeper of the Archives for Oxford University, Brian Twyne (1581-1644) easily fits the archetype of the early modern antiquary, amassing charters and other legal records to defend the university from town interests. He also taught mathematics, flirted with astrology as a career, pored over Allen's famous collection of mathematical books at Gloucester Hall, and pondered the relation of astronomy to his chronological interests. Using Twyne's wide-ranging mathematical manuscript notes, this paper examines Twyne's interest in the history of mathematics, and his critique of Peter Ramus's mathematical practice.

Lewis Ashman: *Colin Maclaurin and the "unexceptionable principles" of Archimedes*

Colin Maclaurin (1698–1746) is most often described simply as a mathematician, and did indeed devote the greater part of his life to mathematical research. A precocious youth, he was appointed Professor of Mathematics at Marischal College, Aberdeen in 1720 and became the occupant of the mathematics chair at Edinburgh five years later, which he held until his death. But Maclaurin's antiquarian interests played a crucial part in his mathematics. In *A treatise of fluxions* (1742) he argues that ancient methods of proof, particularly the "unrivalled" geometrical progression and "unexceptionable principles" of Archimedes, are essential to establishing the veracity of new techniques, such as Isaac Newton's "method of fluxions". Maclaurin's engagement with the methods of the Ancients was central to his approach to mathematics and shaped his responses to contemporary controversies. This paper will explore the prominence and catalytical power Maclaurin accords to the methodology of

Archimedes in his account of the history of mathematics. It aims to highlight the importance of this treatment to Maclaurin's contributions to debates on Newton's calculus in eighteenth century Britain, and to raise questions about the importance of history to the practice of mathematics.

Brigitte Stenhouse, Simon Dumas Primbault, Dalia Deias and Nicolas Michel: *Epistolary Mathematics: Production, negotiation, and circulation of mathematical knowledge in letter correspondence I*

Overview

Recent scholarship has convincingly emphasised the benefits for historians in viewing scientific knowledge as being always in transit, shaped by complex processes of circulation and negotiation. Scientific and mathematical correspondence plays a key role in these processes, enabling communication across cultural, geographical, and sometimes institutional gaps between historical actors. Mobilising rich resources of letters from the 17th to the 19th century, this symposium will bring to the fore the knowledge production witnessed in these sources.

We begin by considering how mathematicians who were geographically and linguistically isolated utilised their correspondence networks in order to situate themselves within an international community of mathematicians. Moreover, we will witness how correspondents could become advocates, aiding in the circulation of novel texts in new locations otherwise closed to the author. In the second talk, we explore how correspondences provide insight into the daily practice of mathematicians in seventeenth-century Europe, and their reliance on informal modes of exposition alongside the emerging hegemony of scientific societies.

We then move from the seventeenth to the nineteenth century. In contrast to geographic and linguistic isolation, as considered earlier, we now investigate how correspondence was used to circumvent social, cultural, institutional and epistemic boundaries.

Access to universities, observatories, and scientific societies was heavily restricted during the nineteenth century, thus creating a gendered barrier to the mathematics produced and exchanged within these spaces. The third talk provides a unique case study of a scientific couple where the husband took on the role of assistant to his more famous wife, mediating epistolary exchanges and thereby enabling her to take part in scientific conversation. Beyond merely bearing witness to which information was circulated and by whom, letter correspondence allows us to witness how texts were read and (mis)understood outside of the communities in which they were produced. The final talk contrasts how controversies and disagreements unfolded through letters and in public, and more specifically how these controversies led to a process of negotiation in which mathematicians attempted to reconcile their differing practices and priorities, in order to reach consensus on the validity of new mathematical results or objects.

Simon Dumas Primbault: *A Trojan Horse across the Channel: On the circulation of Vincenzio Viviani's divinatio through his correspondence with Robert Southwell*

Self-proclaimed last disciple of Galileo, Vincenzio Viviani (1622-1703) strove all his life to become a renowned mathematician. Extolling the supposed purity of Euclid's geometry, he sought to recover the lost knowledge of the Ancients, and fashioned himself a persona as the last heir of the Euclidean tradition. However, Viviani was not appointed primo matematico before 1666 and spent most of his life working as an engineer for the Tuscan Court – a public duty that prevented him from travelling abroad and developing foreign intellectual contacts. Furthermore, having no knowledge of English and a questionable grasp of Latin, his correspondence network shows that his direct sphere of influence did not extend very far from Florence. Clinging to antique geometry in a time of great mathematical progress, Viviani was rather isolated and mostly unknown outside of Italy.

In 1661, Viviani took advantage of his friendship with Robert Southwell to circulate his last publication: a divinatio of Apollonius of Perga's Conics. Back from his Grand Tour, Southwell, who would become president of the Royal Society some thirty years later was then charged with distributing copies of Viviani's book along his way to England. This mission is documented in a series of letters exchanged between the two mathematicians, while Southwell was travelling from Florence back to London, culminating in a glowing letter received by Viviani from Henry Oldenburg, secretary of the Royal Society of London. I will focus on this correspondence between Viviani and his Irish fellow to show how Southwell acted as a trojan horse for Viviani's ancient geometry within the English mathematical network.

Dalia Deias: *The correspondence between Cassini and Oldenburg*

On 22nd May 1672, Giovanni Domenico Cassini (1625-1712), an Italian astronomer working in the Paris Observatory, was elected a fellow of the Royal Society of London. Cassini was proposed by the society's secretary, Henry Oldenburg (1619-1677), with whom he had already corresponded via letter for many years.

In this paper I will focus on the letters between Cassini and Oldenburg, in which they discussed the latest discoveries in astronomy as well as sharing news from Bologna, Paris and London, and display how the contents of their letters evolved over twenty years of correspondence which continued despite cultural differences and political upheaval. I will also situate these letters within the wider context of Cassini's extensive correspondence network, and exhibit how and why this correspondence was important for both the Paris Academy of Sciences and the aforementioned Royal Society.

Brigitte Stenhouse: *Accessing knowledge through polite sociability in 19th-century Britain*

As a nineteenth-century British woman, Mary Somerville (1780-1872) engaged with scientific societies in a manner neither consistent nor straightforward. Women were consciously excluded from such institutions, such as the Royal Society of London and the Royal Astronomical Society, which facilitated invaluable exchanges of knowledge for mathematicians through their published transactions, libraries, and informal social spaces. However, whilst Somerville was never elected a full member of any scientific society, she benefitted from the resources and social networks cultivated in such spaces from as early as 1812.

Using the extensive correspondence held in the Somerville Collection, at the Bodleian Library in Oxford, and the Herschel Papers held at the Royal Society, I will describe how she liberated knowledge from behind the closed doors of learned societies, through the exchange of letters, papers, and astronomical observations. I will focus especially on how Somerville used her husband William Somerville to successfully navigate polite society (here referring to the social interactions between those of high social class, such as the gentry and peerage). With the assistance of her husband, Somerville was able to forge personal acquaintances with her scientific contemporaries in Britain, France, Belgium and Italy, and then mine her expansive network of correspondents for mathematical knowledge. This is epitomised in her correspondence with astronomer John Herschel (1792-1871) during the production of her first book, Mechanism of the Heavens, published in 1831; here we see Somerville as not just a passive receiver of knowledge, but a critical reader producing her own synthesis and interpretation of physical astronomy.

Nicolas Michel: *"Une question de point de vue?": The negotiation of mathematical truth and its modalities in the correspondence of George Henri Halphen*

Upon his untimely death, French mathematician George Henri Halphen (1844-1889), a career military man trained at the Ecole Polytechnique, was renowned and celebrated in France and beyond for his contributions to analysis and algebraic geometry. One of Halphen's earliest contributions, through which he first garnered the attention of mathematicians throughout Europe, was a refutation of a formula conjectured by French geometer Michel Chasles (1793-1880) some ten years earlier. Upon refuting this formula in 1876, Halphen elected to immediately publish his discovery in two brief notes for the Comptes-Rendus of the Paris Académie des Sciences. In a fashion typical for this journal, Halphen only circulated his conclusions, and announced forthcoming extended memoirs on the topic. In the aftermath of these two notes, and whilst these memoirs were being published with extended delays, Halphen engaged in a long three-way epistolary exchange on this topic with two foreign mathematicians, namely Hieronymous Zeuthen (1839-1920) and Hermann Schubert (1848-1911). While both acknowledged Halphen's analytical skills and mathematical virtuosity, they attempted to either restore Chasles' theorem, or preserve some of its significance, from the Frenchman's attacks.

In this talk, we present the Halphen-Zeuthen-Schubert correspondence as an alternative conduit for the circulation of mathematical knowledge, compared with the public exchanges in scientific journals and societies that these three actors had on the same topic. Furthermore, we show how correspondence allowed not only for a discussion of technical, mathematical issues, but also for a complex, contentious negotiation of what should constitute mathematical truth, proof, and epistemically virtuous practice.

Workshop Abstract

Gavin Hitchcock: *"Entrance into All Obscure Secrets"*

A 90-minute workshop on bringing episodes in the history of mathematics to life in the classroom by means of theatre, incorporating a short play set in an ancient Egyptian scribal school.

The workshop provides an opportunity to experience and reflect on the ways that the devices of story, narrative, dialogue, drama, theatre can bring mathematical ideas and history to life in the classroom. We aim to make the case and be inspirational too, by demonstrating theatre in action, involving all participants in the production and enactment of a short pre-scripted play, and then reflecting together on what we have been part of and how it might be used in classrooms, sharing any similar experiences.

The play: The scene is set in a Scribal School in ancient Egypt, about 1800 BCE. Two scribes are seated at a table, each holding quill-pens. There are papyrus scrolls all around, on shelves and tables. They dip the quills periodically into an inkwell and write on papyrus paper. The Head scribe A'H-MOSE orders the junior scribe AMINHOTEP to copy another list of problems and solutions for the students from an old papyrus roll onto a new one. His first task is to start copying out a reference table expressing as sums of unit fractions the quotients of 2 by odd numbers. There are entertaining interchanges over such issues as the rather grand introduction, the importance of showing reverence for the old pharaohs, the sacred nature of the task, the apparently unchanging form of the mathematics over centuries, the importance of copying correctly for posterity, and the rules about whose name gets recorded on the roll – not the junior copier!

Problems worked by AMINHOTEP under the supervision of A'H-MOSE are: (1) How to divide six loaves between the ten men working in the brewery; (2) The unknown added to one fourth of the unknown gives fifteen; what is it? These are RMP Problem 3 and 26. In the play we introduce cultural context, suggesting Problem 26 as arising from an every-day problem about men hired to work on an irrigation canal being given beer for their lunch. During the course of the play, elementary algebraic manipulation is demonstrated and explained: calculating with the unknown number directly until it stands alone equal to a number. The Rule of False Position is also explained, and the play concludes with the solution of a third problem arising from practical needs: the cost of filling a grain bin which has been partially depleted by millstone workers, offset by repayment of a loan to the supervisor of the quarry diggers. The dialogue is lively and accessible, with emotions and humour, aiming to motivate the learning of algebra as well as stimulate an interest in authentic contextual history of algebra.

There are thirteen parts to play, most very small, none requiring memorisation. A narrator, MARIA, introduces the play, and gives commentary. The main dialogue is between A'H-MOSE and AMINHOTEP; but a modern mathematics teacher EMMY, interprets and explains what they are doing. At one point, when AMINHOTEP talks about unknowns, MARIA introduces nine representative scribes/mathematicians from different cultures, who speak one line each, giving the names for 'the unknown' used in equation-solving in their time and context; Maria also explains the meaning of the terms involved. The nine are: EGYPTIAN SCRIBE (2000 BCE), BABYLONIAN SCRIBE (1800 BCE), DIOPHANTUS OF ALEXANDRIA (3rd century), ARABIC MATHEMATICIAN (9th century), INDIAN MATHEMATICIAN (12th century), CHINESE MATHEMATICIAN (13th century), ITALIAN ABBACIST (14th century), EUROPEAN MATHEMATICIAN (15th century), GERMAN COSSIST (16th century).

Individual Talks Abstracts

Francine Abeles: *MacColl's Logic: Christine Ladd-Franklin's Remarkable Opinion*

An outsider in the logic community, Hugh MacColl (1837-1909) eventually achieved considerable recognition of his work in logic in the nineteenth and early twentieth centuries. Together with George Boole, Augustus De Morgan, William Stanley Jevons, Charles S. Peirce, Ernst Schröder, John Venn, Christine Ladd Franklin and others, MacColl considered logic as a calculus represented by the algebra of logic. In an article published in 1889 in *The American Journal of Psychology*, Ladd-Franklin wrote, "Nothing is stranger, in the recent history of Logic in England, than the non-recognition, which has befallen the writings of this author….it seems incredible that English logicians should not have seen that the entire task accomplished by Boole has been accomplished by Maccoll [sic.] with far greater conciseness, simplicity and elegance;". Why did Ladd-Franklin hold this extraordinary opinion of MacColl's work? What may have been her reasons? On what topics in logic did they hold similar ideas? Answers to these questions are the subjects I will explore in this paper.

Amy Ackerberg-Hastings: *Analysis and Synthesis in Robert Simson's The Elements of Euclid (1756)*

In the 18th and 19th centuries, three understandings of the terms 'analysis' and 'synthesis' were particularly influential with the creators and readers of elementary geometry textbooks in Western Europe and North America: as perceived contrasts in styles of mathematical practice in Great Britain and France, as contemporary appeals to ancient methods of proof, and as approaches to mathematics education. One especially influential textbook arose from the attempt by University of Glasgow mathematics professor Robert Simson to restore Euclid's text, which appeared in 1756 as The Elements of Euclid, in simultaneous English and Latin versions. The talk will explore what we can learn about the book's preparation and reception by examining it through the lenses of analysis and synthesis. The topic inherently evokes all three of the conference themes, people, places, and practices.

Marion W. Alexander: *Euler's Construction of a Continued Fraction for e using Differential Equations*

One of the silver linings of the pandemic in 2020 is that the ARITHMOS reading group went virtual, allowing several of us to participate when physical distance and time constraints before Covid would have prevented it. My interest in the history of continued fractions led me to ask the group if we could read Euler's paper, "De Fractionibus Continuis Dissertatio," (E71), (written in 1737, published in 1744). This paper is more often cited as where Euler proves that e is irrational. Euler's "proof" in that paper flummoxed the ARITHMOS reading group last fall; after doing a little research, we found several authors had been equally stymied by it. Fred Rickey suggested we look to Rosanna Cretney's 2014 paper, "The origins of Euler's early work on continued fractions." (Historia Mathematica 41 (2014), 139-156, for some guidance. There, Cretney shows how Euler (in a letter to Goldbach (R729) in 1731) may have devised a technique using Daniel Bernoulli's treatment of Ricatti equations in his 1724 book, Excertationes. We decided to repeat Cretney's method on the slightly different Ricatti equation Euler was using in E71. The result seems to be a much more convincing proof of his presentation of an infinite simple (regular) continued fraction expansion for e, perhaps lessening the doubt expressed by some that he had actually proved e irrational in E71, in the first place.

R.A Bailey: *Latin squares at Rothamsted Experimental Station in the time of Fisher and Yates*

In the 1920s, R. A. Fisher, at Rothamsted Experimental Station in Harpenden, recommended Latin squares for agricultural crop experiments. Frank Yates became his assistant in 1931, and head of Statistics when Fisher left two years later.

In the early 1930s Fisher and Yates developed sets of mutually orthogonal Latin squares (MOLS). They also introduced the duals of finite Abelian groups for the construction of designs for factorial experiments.

In 1938, Fisher and Yates published their famous "Statistical Tables for Re- search Workers" (STRW). In the same year, R. C. Bose published what became the standard method of constructing both factorial designs and mutually orthogonal Latin squares, using finite fields.

In 1942, Fisher pointed out that the set of 9×9 MOLS in STRW is not isomorphic to that constructed by Bose. The explanation in STRW shows how they are constructed from an elementary Abelian group.

Christopher Baltus: *Leading to Poncelet: A story of collinear points*

Even for a highly original work, Jean-Victor Poncelet's Traité des propriétés projectives des figures, of 1822, previous work prepared the ground. The claim of the talk is that the prevalence of problems and propositions in which collinear points (or concurrent lines) are assumed or demonstrated is a good measure of the projective character of work that follows. Euclid, Apollonius, Ptolemy, Pappus, Desargues, Monge, L. Carnot, and C. J. Brianchon all have roles in the story, with special attention to the first decade of the nineteenth century.

Michael J. Barany: *Bibliographic globalization in the history and historiography of modern mathematics*

The problem of comprehensive and timely mathematical bibliography was at the forefront of mathematicians' international discussions at the turn of the twentieth century. By mid-century, two dominant enterprises provided the connective infrastructure that would hold together the mathematical literature for the remainder of the century and set the terms for new bibliographic undertakings up to the present: the Europe-based Zentralblatt für Mathematik und ihre Grenzgebiete and the America-based Mathematical Reviews. These abstracting journals and the large mathematical and publishing networks that contributed to them together created a meaningfully global mathematical literature, changing in the process how mathematicians organized their research communities and their ideas alike.

My short presentation will outline a new program of research that combines archival analysis with database methods to examine how mathematicians and publishers synthesized a coherent literature, and will present some preliminary findings from this investigation. The research takes advantage of recent archive-based historical work on the early histories of the Zentralblatt and Mathematical Reviews as well as large-scale digitization efforts reflected in the zbMATH and MathSciNet online databases. Creating a coherent literature included developing systems of classification—initially ad hoc and variable tables of contents and cumulative indices—that eventually became the formal, hierarchical Mathematics Subject Classification shared between the Zentralblatt and Mathematical Reviews. Editors and reviewers collaborated to articulate, enforce, and adapt a view of the whole of mathematics that responded both piecemeal and macroscopically to a discipline whose personal, geographical, and intellectual scales expanded dramatically in the latter half of the twentieth century. A view of mathematical globalization that emphasizes the distributed labour and effects of producing an integrated bibliographic infrastructure has consequences both for historical understandings of modern mathematics and for historiographical approaches to accounting for research and institutions in mathematicians' global era. These include hypotheses relating mathematical research infrastructures to organizational forms, career patterns, conceptual structures, and philosophical structuralism connected with modern formations in the discipline.

Philip Beeley: *'Dr Gregory's scheme'. Reforming mathematics at the English universities around 1700*

In recent years there has been a considerable debate over the nature of mathematical instruction at the English universities of Oxford and Cambridge during the second half of the seventeenth century, focusing on a central question: how was mathematics taught and to what standard? One of the problems arising from this debate is the apparent disparity between the content of lectures, many of which had survived in book or manuscript, and the texts undergraduates were required to read by their college tutors. Contemporary sources indicate that general compendia such as Pierre Gautruche's Mathematicae totius were widely used for college teaching, but this fact is hard to square with the sophisticated content of, for example, lectures presented by the Savilian professors at Oxford. The talk will seek to provide answers by approaching the topic from a new viewpoint. Recognition of the deficits in mathematics instruction in the universities combined with increasing competition through knights' academies both at home and abroad motivated a major reform around 1700. This reform, primarily associated with the then Savilian professor of astronomy, David Gregory, encompassed course structure and content and saw the introduction of assessment based on examinations. It will be argued that these changes not only throw light on previous teaching methods but also set the scene for the way mathematics was taught in the universities over the course of the following century.

Kristín Bjarnadóttir: *Nordic Cooperation on Modern Mathematics at Primary Level 1960–1968*

In November 1959, a seminar on new thinking in school mathematics was held in Royaumont, France. Representatives from all member countries of OEEC (later OECD) and Yugoslavia attended the meeting except Portugal, Spain and Iceland. Participants from Denmark, Norway and Sweden agreed upon organizing Nordic cooperation on reform of mathematics teaching. Finland, not a member of OEEC, was invited to join. The object of this study is to analyse this cooperation and its influences in Iceland, which was not invited, by examining original archived documents and reports. The project had a great impact on the evolvement of mathematics in Iceland, a new-born state in the mid-1900s.

The issue was taken up in the Nordic Council. The Nordic Committee for Modernizing Mathematics Teaching, Nordiska kommittén for modernisering af matematikundervisningen, NKMM, was set up under its Culture Commission. Each country appointed four persons to the committee. Its members were mathematicians and mathematics teachers, or school administrators. The programme for the Nordic reform was to analyse the situation found within each country, to work out preliminary and revised curriculum plans, and write experimental texts.

The committee appointed writing teams. Its main object was teaching of grades 7–12. However, it was decided to handle mathematics courses throughout the school. The committee contacted for that purpose extra experts for the first six grades. Support was gained from OEEC and the Nordic Council. Other costs were divided between the four countries where Sweden with the largest population was to pay the most. Joint Nordic manuscripts were planned. Persons from each country would translate and adapt the joint publications to each language.

Examination of archive documents about the activities of the committee reveals that many obstacles hindered the realization of the cooperation in the sense to produce material in common. There were language hindrances, the Finns could not use a Scandinavian language as a working language, where others could speak their mother tongue. There were different school systems in the four countries, and the school systems were undergoing reforms, mainly in the direction to move from seven to nine years' compulsory school, and the member of the writing teams had different opinions as to in which direction the reform was to head. At the end, very little of the published material was in common to two or more countries.

This research focuses on the primary-level cooperation. Norway and Sweden took their own direction. Denmark and Finland cooperated on grades one and two. Thereafter the Danish Ms. Agnete Bundgaard produced a highly theoretical syllabus which she ran in Denmark, Iceland and Greenland. The result raises questions about curriculum reforms, leadership and contents.

Mónica Blanco: *Pedro Padilla and his Military Course of Mathematics (1753-56): Teaching Higher Geometry in Eighteenth-Century Spain*

Toward the end of 1750, an Academy of Mathematics was established within the Military Academy of Royal Guards of Madrid that was managed by Pedro Padilla (1724-1807?) until it closed in 1760. In 1753, Padilla authored and published his Military Course of Mathematics (1753-1756) for the specific use by this academy. Of the twenty mathematical treatises that Padilla originally intended to develop, only the first five would be published in the end (in four volumes): (1) Ordinary arithmetic; (2) Elementary (or Euclidean) geometry; (3) Elementary algebra; (4) Higher geometry, or geometry of curves, and (5) Differential and integral calculus, or the method of fluxions.

The fourth treatise is one of the earliest educational books on analytic geometry to be written in Spanish, a work that has been little studied so far. In this treatise Padilla dealt with the geometrical construction of equations, conic sections and the resolution of a wide variety of geometrical problems by means of algebra. The analysis of this treatise contributes evidences of the introduction of analytical geometry in Spain, in particular, within the Spanish system of military education in the first half of the eighteenth century.

Viktor Blåsjö: *Algebraic versus geometric thought and expression in the early calculus*

The language of the early calculus was much more geometrical than the analytic and algebraic style that was pioneered by Euler and still dominates today. The use of trigonometric expressions such as sin(x) in a calculus context were unprecedented before Euler, who indeed explicitly claims priority for this development. Calculus formulas involving log(x) were never used by Newton and only twice by Leibniz in all his publications. Instead, early practitioners of the calculus used geometrical paraphrases. They also often insisted on dimensionally balanced equations, such as writing ydy=adx rather than ydy=dx (which was considered inappropriate since it compares an area to a length, or a rectangle to a line segment).

Interestingly, however, the geometrical paradigm of the early calculus was not due to mere conservatism or failure to conceive alternative approaches. On the contrary, in a few telling places in early private notes, Leibniz and Newton in fact use log(x) and cos(x) in calculus formulas. Early practitioners of the calculus also often disregard dimensional homogeneity in private in a much more casual way than in their publications. They also showed no signs of being constrained in their thinking when the occasion called for formal, non-geometrical applications of calculus principles such as applying the fundamental theorem or partial differentiation abstractly n situations that are not visualisable in terms of tangents and areas. This suggests that the adherence to the geometrical mode of expression in the early calculus was a deliberate choice selected with full awareness of the analytic alternative. What compelled these early calculus practitioners to make this choice? Besides lip service to classical foundations, I argue that the geometrical paradigm had genuine merits as an intuition-boosting heuristic strategy.

Piotr Błaszczyk: *Euler's expansion of cos x function*

1. We provide a self-contained introduction to the following concepts: infinitely small and infinitely large numbers, relation x is infinitely close to y, $x \approx y$, operation $\sum_{i=1}^{K} a_i$, where K is infinitely large number. The exposition is based on the ultrapower construction and does not require any previous knowledge of mathematical logic; see [1], and [2]. With these tools, we aim to analyze [3, §132-134]. Throughout these short paragraphs, Euler derives the most famous mathematical formula: $e^{ix} = \cos x + i \sin x$. It is based on the expansion of $\sin x$ and $\cos x$, as well as e^x into a series. We focus on how Euler expands the $\cos x$ function.
2. Euler explicitly refers to infinitely small and infinitely large numbers. We explain these concepts within the framework of non-Archimedean field, and show that [4, ch. 3] explicitly discusses some rules of an ordered field, specifically these referring to negation of the Archimedean property. We also show that $\sin x$ and $\cos x$ in a non-Archimedean field context are specifically defined functions $\sin * x$ and $\cos * x$.
   Euler also employs infinite sums, nevertheless, they are not linked to the concept of limit of sequence. While [3] usually refers to these sums by the abbreviation sign &c., [4, §107] reveals the real meaning of that operation, namely
   $$1 + a^1 + a^2 + a^3 + \&c. = 1 + a^1 + a^2 + a^3 + \cdots + a^k = \frac{1-a^{K+1}}{1-a} = \frac{1}{1-a}$$
   where K is infinitely large number and $|a| < 1$. We interpret Euler finite sums by hyperfinite sums of nonstandard analysis.
3. At the very beginning of [3, §134] Euler adopts identities $\sin z = z$ and $\cos z = z$. In our interpretation, they are explained by the *is infinitely close to* relation, namely: $\sin z = z, \cos z = 1$.
   Then Euler's claim
   $$\cos nz = \frac{(\cos z + i \sin z)^n + (\cos z - i \sin z)^n}{2},$$
   when is a „finitely large number", we will interpret as

$$\cos * Kz = \frac{(\cos * z + i \sin * z)^K + (\cos * z - i \sin * z)^K}{2},$$

for infinitely large K.

Finally, we show that instead of Euler's identity

$$\cos nz = 1 - \frac{v^2}{1 \cdot 2} + \frac{v^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{v^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \&c,$$

we can show that the following relation obtains

$$\cos * nz \approx 1 - \frac{v^2}{1 \cdot 2} + \frac{v^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{v^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \&c,$$

where &c. we interpret as the hyperfinite sum.

Eugene Boman: *A Story of Real Analysis; An OER Textbook*

The modern definition of continuity is so non-intuitive that its necessity is typically lost on students who have rarely dealt with discontinuous functions in any substantial way. Similarly, the need for the limit definition of the derivative is lost on students who have always thought of the derivative as "the slope of the tangent line". It is only after the inherent difficulties of Leibniz's differentials (or Newton's ultimate ratios) have become clear that the necessity of these highly non-intuitive definitions can be justified.

Real Analysis is frequently taught in a manner which highlights the logical/topological structure of the real numbers. That is, students are first taught to build the real numbers from the rationals. Then the topology of the reals is studied, with particular attention to those properties (e.g., the Least Upper Bound property), and concepts (e.g., continuity) which support the foundations of Calculus. This is an entirely valid approach. After all, every mathematician needs to understand the logical underpinnings of modern analysis.

But, despite its appeal to trained mathematicians, this approach is far from optimal, pedagogically. Since analysis evolved from attempts to address the foundational issues posed by the Calculus of Newton and Leibniz we suggest that an historical (chrono-logical) approach has certain pedagogical advantages. To be sure this approach is not as "clean" as the traditional approach but we don't really see that as a flaw. It is only after a mature theory has been developed that a clean, logical construction is possible.

But students in real analysis are mathematicians-in-training. They are poorly served if they are left with the impression that mathematics is developed cleanly, starting with the basic principles, rather than by the messy, sometimes chaotic process that we see in its history. Taking an historical perspective has the pedagogical.

Robert E. Bradley: *Convergence and Divergence in the Works of François-Joseph Servois*

François-Joseph Servois (1768-1847) was a proponent of Lagrange's scheme of providing a foundation for differential calculus based on power series. His most influential work was a paper on differential operators: "*Essai sur un nouveau mode d'exposition des principes du calcul différentiel*" (1814). Nowhere in the free-wheeling manipulation of formal power series which he undertook in this essay did Servois consider questions of series convergence. However, a few years later in his "*Mémoire sur les quadratures*" (1817), Servois displayed a keen understanding of issues of divergence and convergence. Along the way, he delivered a stinging rebuke of the use of divergent series in calculation. In this talk, we will examine both sides of Servois' mathematical character and attempt to reconcile the formalist Servois with hard-nosed analytical alter-ego.

Bruce Burdick: *Three Generations of Printers and Almanac Authors in Seventeenth Century Mexico*

Enrico Martínez, Juan Ruiz, and Feliciana Ruiz represent three generations of printers in Mexico City spanning a period from the late sixteenth century until 1676. The art of printing was passed down from one to another.

The owner of a printing press sought to publish an almanac every year because these were popular items and earned money for the printer. Most printers needed to find an author with the mathematical competence for such an undertaking but these three wrote their almanacs themselves. It is plausible that the mathematical skill required was also passed down from one to another.

Almanacs supplied their readers with an account of what sign the moon was in for each day of the year as well as tracking the planets and predicting eclipses. The role of astrology in the lives of the people of this time made these almanacs essential reference works.

We will discuss the Reportorio de los Tiempos of 1606 by Enrico Martínez, survey what is known about the almanacs of his son, Juan Ruiz, and show the request by the latter's daughter, Feliciana Ruiz, asking permission from the inquisition to publish the almanac for 1676.

Joao Caramalho Domingues: *Geometry and analysis in Jos Anast cio da Cunha's fuuionary calculus*

It is well known that, as the 18th century progressed, the calculus moved away from its geometrical origins, with Euler, and later Lagrange, aspiring to turn it into 'pure analysis'.

The Portuguese mathematician Jos Anastácio da Cunha (1744–1787) has some claim to fame for, among other reasons, giving an original definition of fluxion which Youschkevitch described as the first 'rigorous *analytical* definition of differential' (my emphasis). However, it is well known that Cunha was very critical of Euler's faith in analysis; and Gomes Teixeira pointed out the role of 'geometrical intuition' in Cunha's calculus.

Given all this, should we classify Cunha's functionary calculus as an example of late 18th-century analysis or as a survival of geometrical tendencies (it would be far from unique)? Or could it fall outside these classifications? This issue will be addressed, drawing on Cunha's opinions on philosophy of mathematics and on an interesting observation from a friend of his about this definition of fluxion.

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Lawrence D'Antonio: *Clairaut Repeals the Inverse Square Law*

In the 1680s Newton famously tested his inverse square law of gravity on the problem of predicting the orbit of the Moon, but failed. His computation of the precession of the lunar apsides gave only half of the observed value. The problem of the Moon's motion lay unsolved but was taken up again by Clairaut, Euler, and d'Alembert in the late 1740s. At a meeting of the Paris Academy of Sciences in 1747, Clairaut announced that he and Euler had attempted to calculate the rotation of the apsides, but as had Newton, they found only half of the observed value. To address this failure, Clairaut proposed the replacement of the inverse square law with a law having both an inverse square and inverse fourth power term. In this talk we will consider Clairaut's failed attempt to predict the motion of the apsides, his abandonment of the inverse square law, the severe reaction of the Newtonians in the Paris Academy (especially that of Count Buffon), and Clairaut's successful computation that correctly predicted the rotation of the apsides and saved the inverse square law.

Gregg De Young: *From Proclus to Albertus Magnus: Transmission of a collection of "proofs" of Euclid's postulates*

This paper outlines the historical transmission of a set of "proofs" for Euclid's postulates that circulated over several centuries in several mathematical communities in several different languages. These proofs are first found in the Greek commentary on book I of the Elements by Proclus (died about 485 CE), who brought together several earlier "proofs" of some of Euclidean postulates and apparently added other "proofs" of his own devising. These "proofs" reappear in the early Arabic commentary of al-Nayrīzī (died early 3rd / 9th century), where they are attributed to the Hellenistic commentator, Simplicius (died after 533 CE). We next encounter the "proofs" some three centuries later in an Arabic recension (Iṣlāḥ) of the Elements by Athīr al-Dīn al-Abharī (died 663 / 1264). A newly discovered independent Arabic discussion of these proofs by an unknown author features a reformatting these "proofs" and includes an apparently new "proof" of the parallel lines postulate, presumably by the unknown author himself. Quṭb al-Dīn al-Shīrāzī (died 710 / 1311) inserted a Persian rendition of these "demonstrations" into his Persian translation / edition of Naṣīr al-Dīn al-Ṭūsī's Taḥrīr of the Elements which he included in his philosophical compendium, Durrat al-Tāj. These "proofs" were also transmitted into Latin, appearing in the commentary attributed to "Annaritius". Their final appearance on the stage of history was in the early Latin commentary on the Elements attributed to Albertus Magnus (died 1280 CE). The history of this collection of "demonstrations" offers a microcosmic view of mathematics in motion -- the way a specific set of mathematical ideas was transmitted across time and across linguistic and geographical boundaries. It also reveals the changes, some only formal and others substantive, both in text and diagrams, that accompanied these temporal and geographical peregrinations.

Anne Duffee: *Ghosts of Mathematics Past*

How do paradigms form, and shift, in mathematics? Using Thomas Kuhn's The Structure of Scientific Revolutions as inspiration, I explore a handful of "revolutionary" moments in the history of mathematics. Through this exploration, I endeavor to show how mathematics, as a discipline, changes throughout time: in which old paradigms of mathematics seem to persist within mathematical culture, in a way that seems unique in relation to the normal sciences.

I consider three decisive moments in the history of mathematics: Descartes' solution to the problem of nonhomogeneity; Dedekind's construction of the reals; and Hilbert's axiomatization of geometry (together with some of his commentary on the relation of concepts and axioms). Many historical shifts in mathematical disciplines occur because of what I will term "conceptual obstacles.". I argue that new paradigms in mathematics do not solve historical problems, but instead constitute new conceptual frameworks in which such problems can no longer exist. In all three of the historical moments I will cite, this can be demonstrated. The issue of incommensurability of heterodimensional objects (and beyond this, the problem of what a "four dimensional" object could even be), within the problem of nonhomogeneity is "solved" by abstraction from geometric intuition. That density is insufficient for continuity is "solved" by the completion of the rationals, itself a fairly profound abstraction from what we would intuitively call number. And lastly, the myriad problems of the foundation of Euclidean geometry are similarly "solved" by the creation of a parallel axiomatic system, albeit one in which our primitive concepts of line, point, etc., have no inherent meaning.

In all three historical instances, I claim that the "solution" to the conceptual problem is in fact a creation of a new framework in which the traditional problem no longer makes sense, and that the new framework, or paradigm, is one in which certain axioms, either implicit or explicit, have subtly redefined the foundational concepts of the discipline in order that the old problems no longer obtain. To paraphrase Hilbert, Euclid's "point" and his own "point" are entirely separate concepts. And so in the creation of each new paradigm, it is the structure, or the set of axioms, itself that is foundational, rather than the concepts that we might believe are common to both.

I conclude by examining the way in which ghosts of old paradigms seem to persist, especially within realms of intuitive argumentation and mathematical pedagogy. Whereas in scientific paradigm shifts, the conceptual framework of the paradigm is left wholly behind, in mathematics, ghosts remain. Despite the construction of new foundational material, appeals to intuition remain within mathematics, both in lower education as well as in certain informal arguments, especially with regard to individual concepts like point, number, dimension.

Adam Dunn: *From the Local to the Global: The Evolution of Statistical Thought and Practice in the Eighteenth Century*

This paper will explore a two-pronged approach to the history of statistics in the eighteenth century. First, it will analyse the changing methods of data collection used by practitioners of statistics, especially in demography and the early census. It will argue that changing practices and methodologies, such as the use of wider networks of informants, desire for more empirical and numerical information and the need for more efficient ways of gathering and analysing this information, created a need for a more mathematical, precise and scientific method of collecting statistical data. This is a trend that becomes increasingly visible in the work of early census takers, amateur demographers and statisticians. The movement towards more efficient practices of data collection served to highlight a glaring hole in the old forms of statistical practice. Second, it will analyse the changing definitions and methodological approaches of statisticians throughout the eighteenth century. It will illustrate how new practices, definitions and theories were driven through developing statistical networks that spread across Europe, defined not by single geographical spaces but by a wider 'pan-European' transnational space. This space opened statistical thought to a myriad of new ideas that helped to transform it from a purely descriptive enterprise to a fully-fledged mathematical science. It was not controlled by preconceived notions of state, nation or border. Instead, there flourished a wider, mathematical and scientific community, which did not discriminate on the basis of expertise or nationality, that spread new and ever more complex ideas about statistics, incorporating a range of interests (from the mathematical to the economic to the social). The paper argues that local practical experimentation drove higher level theoretical developments in the history of statistics, as transnational statistical communities negotiated and renegotiated the practical and theoretical implications of data collection and analysis. It will illustrate how cross-border connections and communities formed integral components in this evolution of statistical thought. Taking examples in Scotland, Germany, Switzerland and France, this talk will try and demonstrate how the changing practices of statistics were brought about through diverse transnational communities who were experimenting with new methods in their own backyards.

Abe Edwards: *All the Lines: Cavalieri's Geometria indivisibilibus and the End of the Jesuats*

During the first half of the seventeenth century, Italian mathematicians and scientists developed several important ideas that laid the foundations for calculus. Nevertheless, many eventually found themselves on the wrong side of the counter-reformation that flowed out of the Thirty-Years War. Galileo was placed under house arrest in 1633, and the shadow of the Vatican loomed over the Italian countryside for decades to come.

In 1635, Bonaventura Cavalieri published *Geometria indivisibilibus continuorum nova quadam ratione promota* which represents an important bridge between the Archimedean "method of exhaustion" and the calculus which Newton and others were to develop only a few decades later. At the time of its publication, Cavalieri (best known for his namesake principle in geometry) was a member of the Jesuati religious order, and served at the Church of Santa Maria della Mascarella in Bologna. His work attracted both admiration, and attack, from mathematicians. His ideas about indivisible quantities made him the target of rival religious groups who saw his work as a direct challenge to Scholasticism – the predominate philosophical system of the time.

In 1668, Pope Clement IX commanded that the Jesuati order be forever disbanded. His decision was motivated, in part, by a desire to crush all those who taught Cavalieri's heretical ideas about indivisibles. In this talk, we will investigate the cultural forces that shaped intellectual discourse in seventeenth century Italy, and the influence of those forces on the history of calculus. We will see why Cavalieri's ideas were controversial, both mathematically and philosophically.

Four hundred years later, the lessons of these events still echo a warning for politicians, people of faith, and mathematicians.

Kenneth Falconer: *John Couch Adams – From Neptune to St Andrews*

John Couch Adams is best known for his controversial 'discovery' of the planet Neptune. He was an undergraduate at Cambridge, where he became Senior Wrangler and where he spent most of his career, as a college fellow, as Lowndean Professor of Astronomy and Geometry and as Director of the Cambridge Observatory. However, for a short time he was Regius Professor of Mathematics at St Andrews. This talk concentrates on Adams' association with St Andrews, reflecting on some remarkable correspondence with Brewster, Stokes and others which has recently come to light.

Craig Fraser: *Constantin Carathéodory and the Theory of Canonical Transformations*

A common form of history of mathematics is to examine the development of a part of the subject through history. Thomas Hankins (1979, 11) in an article "In defence of biography: The use of biography in the history of science" suggests that biography "is not the proper mode for describing the development of science through time." Nevertheless, a biographical focus can illuminate an internalist survey of a mathematical subject. The present paper examines the history of a modern mathematical subject—the theory of canonical transformations in Hamilton-Jacobi theory—through the biographical lens of one prominent researcher. The historiographical claim is that such an approach enhances the usual focus on the development of concepts, results and methods.

Constantin Carathéodory (1873-1950) was an historical figure with several sides. Coming from a Greek cosmopolitan background he played a prominent role in German mathematics from the early years of the century to his death in 1950. Among his contributions to Hamilton- Jacobi theory was his 1935 book Variationsrechung und Partielle Differentialgleichungen erster Ordnung. Carathéodory was also interested in the history of mathematics and made some contributions to the history of the calculus of variations.

The theory of canonical transformation was invented by researchers working in celestial mechanics, mathematical analysis and quantum physics. Carathéodory 's 1935 book reflected the analyst's perspective on the theory. He was in contact with both physicists and mathematicians who engaged with the study of canonical transformations. Biographical details concerning his place in the research milieu of the period provide insight into the distinctive imprint he made on the mathematical subject.

Pedro Freitas and Jorge Nuno Silva: *On the first Portuguese book on mathematics (Tratado da Practica Darismetyca (1519))*

From the end of the Middle Ages, several treatises on arithmetic appear in Europe. Primarily focused on commercial mathematics, they played a major role in preparing the next generations for economic activity never seen before. The typical content of these works includes the Hindu-Arabic decimal numbering system, and the associated algorithms, several of which we still use, multiple calculation techniques applied to commerce, such as the rule of three, and recreational problems.

In Portugal, the first printed book of mathematics, the *Tratado da Pratica Darismetyca* (1519), by Gaspar Nicolas, is part of this tradition. As Nicolas claims, he drew on a late fifteenth-century work by Luca Pacioli, an Italian mathematician, to compose his text, which had a dozen editions, the last of which in the eighteenth century, which attests to its great influence and acceptance. To celebrate the 500th anniversary of its appearance, the speakers prepared a modern version of the full text, published by Fundação Calouste Gulbenkian, which will be surveyed in this talk.

The *Tratado* consists on teachings of commercial mathematics, mainly arithmetic. Nicolas introduces the numbers, the basic operations (addition, subtraction, multiplication, division, square roots), illustrated with many examples. Then, he gives some rules useful for commerce (for example, the rule of companies and the rule of three). A collection of recreational problems and a chapter on silver alloys complete the book.

The collection of recreational problems is very interesting, as we recognise some problems in the tradition of Alcuin, Fibonacci and Pacioli, among others. We will clarify this statement with particular examples during our talk.

Michael Friedman: *A geographical seclusion? On the Italian "school" of braids of Oscar Chisini between the 1930s and 1950s*

In 1933 the Italian mathematician Oscar Chisini (1889 – 1967) published his paper "A Suggestive Real Representation for Plane Algebraic Curves" ("Una suggestiva rappresentazione reale per le curve algebriche piane"), in which he suggested to use material models of braids in order to present a new mode of visualization of singular plane curves. Though during the years later, the research of singular plane curves (or braids) using material models of strings was not further researched, in 1950 a student of Chisini, Modesto Dedò (1914 – 1991), continued Chisini's research and developed a new notation for braids, based on these material models. This research has caused a renewed interest in the research of braids with the help of Dedò's notation, and as a result during the 1950s a group of Chisini's students (Carlo Felice Manara, Cesarina Tibiletti Marchionna, Ermanno Marchionna and of course Modesto Dedò) continued the research of braids and singular plane curves. In that sense once can speak of an Italian "school" – mainly of Oscar Chisini, which was formed during the 1940s and mainly during the 1950's.

The question that rises is whether this school had any influence on the mathematical research of braids inside and outside of Italy, and whether it had any influence on other areas of the mathematical research, in the domain from which it initially developed: algebraic geometry. The answer is rather a negative one: starting the end of the 1950s, all of the students of Chisini stopped the mathematical research of braids and turned to other research directions. The talk aims to uncover the reasons for this abandonment: was the research not mathematically fruitful enough or too complicated to master? Were there other mathematical traditions of the research of knot and braid theory, which were more powerful? Indeed, a possible reason for the disappearance of Chisini's school would be the fact that the methods that Emil Artin developed in 1925-6 in his paper "Theory of braids" ("Theorie der Zöpfe") were already widely accepted. But were there other reasons, dependent on the re-writing project of algebraic geometry during the 1950s? Or did the rise of fascism and the political situation in Italy during the 1930s and the 1940s continue to influence the image of the mathematical community in Italy during the 1950s?

Eduardo N. Giovannini : *From Euclidean to Hilbertian practice: the theories of plane area*

The aim of this talk is to analyze the theory of plane area developed by Euclid in the Elements and its modern reinterpretation in Hilbert's influential monograph Foundations of Geometry (1899). Particular attention will be bestowed upon the role that two specific principles play in these theories, namely the famous Common Notion 5 ("The whole is greater than the part") and the geometrical proposition known as De Zolt's postulate ("If a polygon is divided into polygonal parts in any given way, then the union of all but one of these parts is not equivalent to the given polygon"). The latter postulate was the main focus of a geometrical discussion on the foundations of the theory of plane area, at the end of the nineteenth century and the beginning of the twentieth century. The fundamental role that this geometrical proposition played in the development of the theory of area was explicitly stressed in the context of this foundational discussion: since this proposition excluded the possibility that a polygon could have lesser area than itself, it was essential for the introduction of a relation of (strict) order for polygonal areas. De Zolt's postulate was then usually taken as the mathematically precise and purely geometrical formulation, for the case of polygonal areas, of Euclid's Common Notion 5 in the Elements.

During this period, the foundational discussions in connection with De Zolt's postulate concerned mainly three general issues. The first one was related to its logical status within the theory of plane area. By the end of the nineteenth century, the usual standpoint in elementary geometry textbooks was to include De Zolt's postulate as an axiom. However, the fact that several proofs were already achieved triggered the question whether or not De Zolt's postulate should be adopted as a geometrical axiom. A second issue concerned the necessary and sufficient conditions for proving it. In particular, the quest for a purely geometrical proof of De Zolt's postulate, as well as the elucidation of the minimal assumptions required for such a geometrical proof, were significant challenges during this early period. The third issue consisted in providing a proof for the fact that plane polygons determine a class of geometrical magnitudes. This geometrical problem was connected to the modern axiomatic investigations of the concept of magnitude.

Firstly, I will be argued that an adequate elucidation of how these two principles are conceptually related in the theories of Euclid and Hilbert is highly relevant for a better understanding of the respective geometrical practices. Secondly, I will claim that these conceptual relations unveil interesting issues between the two main contemporary approaches to the study of area. of plane rectilinear figures, i.e., the geometrical approach consisting in the geometrical theory of equivalence and the metrical approach based on the notion of measure of area. Finally, I will analyze how the different strategies commonly used to provide a proof of De Zolt's postulate raise interesting issues for the current discussions on the "purity of method" in mathematical practice.

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Stefano Gulizia: *The mathematical network of Nicolaus Granius (1569-1631): Mathesis, Copernicanism and Scribal Technology in Helmstedt*

This paper takes a fresh look at the mathematical book trade in Northern Germany, between 1570 and the 1630s, by taking as a reference point the printed collection of a Swedish itinerant professor, Nicolaus Andreae Granius (d. 1631), who taught physics in Helmstedt and annotated more than four hundred scientific editions. It shows that these transactions were fueled by transfer and communication as much as by warfare and confessional struggle, and it suggests that our available mathematical notebooks and marginalia should be evaluated as a "special genre" in the history of science, both as teaching tools and objects of courtly display, which may be characterized as a point of crystallization for a truly interdisciplinary historical epistemology. A particular aspect of interest here is the mathematical education that Scandinavian clerici vagantes received in diverse academic locales such as Prague, Rostock, and Helmstedt. First, I offer a sketch of the main textbooks acquired by Granius and printed either in the Netherlands or in the Venetian region, including Giambattista Benedetti's systematically anti-Aristotelian Diversarum speculationum… liber (1585). Second, thanks to a new attribution to Granius of Copernican annotations preserved at the HAB, I debate whether our historiography of a rift between realism and conventionalism is sustainable. Finally, I reassess the influence of Ramist mathematics in Helmstedt by using Granius's theoretical notes as well as his distinctive tables and diagrams.

Geoff Hagopian: *Visualizing Napier's Rules in Trigonometry - A Vignette*

John Napier (1550-1617) created a mnemonic for reducing 10 equations relating the sides and angles of a spherical triangle down to 2 equations, which became known as \Napier's Rules." My uncle recently shared with me his 1950's HS trigonometry text. I was pleasantly surprised to discover that this text not only provides a complete derivation of the 10 equations for solving spherical triangles and how Napier's Rules simplify these, but also a paper folding exercise for visualizing key components of the formulas. I invite you to participate in this paper- folding visualization for this talk. The original publication date for this text, Modern Course in Trigonometry was 1947, just two years after the end of WWII. The two authors were Alfred Hooper, a British math teacher and headmaster who became an officer in the R.A.F. and also wrote The River Mathematics and Makers of Mathematics, and author Alice L. Griswold, a New York City HS math teacher whose prestige and expertise no doubt gave this more ambitious HS text credibility in America. Hooper and Griswold's approach to introducing spherical geometry is rigorous, while employing a hands-on paper-folding construction to help
make these concepts more concrete, instructing students at the start to "\Make a scale drawing, tracing all lines on sides of a heavy paper." This figure when properly folded illustrates a 3-dimensional section of the sphere.

Students are then instructed to fold this into a 3-D spherical polyhedron After folding, spherical 4ABC, and the right triangles XY Z and X1Y1Z1 are formed, which triangles are parallel 1 to the planes tangent to the sphere at the points A and B, respectively, and so that spherical \A = \XY Z and spherical \B = \X0Y 0Z0.

Robert Moritz' 1913 A Textbook on Spherical Trigonometry presented what he claimed to be a novel proof of Napier's Rules{but, for better or worse (and I think for the worse), since Hooper and Griswold's revival (which also features a proof of Napier's Rules), spherical trigonometry has disappeared from the curriculum, even at the baccalaureate level. Perhaps it's time for a revival!

Gavin Hitchcock: *Corrupt Land Inspectors: Solving equations with picture-language in ancient Mesopotamia, a dialogue*

This presentation, involving one or two other participants, combines three aims: it may be taken as an entertaining piece of mathematical theatre; as a dramatic evocation of ancient Mesopotamian cultural perceptions of mathematics and mathematical practice; and as an illustration of the power of theatre (and history of mathematics) to motivate learners in algebra.

The play takes place in a Mesopotamian Scribal School, nearly four thousand years ago. There is a sand tray for diagrams, and a number of clay tablets, both hard- baked ones, and moist ones for imprinting with styluses. There is a bitumen-lined bin for recycling tablets. The student scribe, Ea-shar-ili, complains to the Head Scribe,
Ku-ningal, that tax-collectors are defrauding the poorest farmers by using, as tax- bracket standard, the length of fence around a field. The Head Scribe has used this to motivate a lesson on finding length and width of a field, given the semi-perimeter
and the area. Now, in a follow-up lesson, they review this form of problem, and go on to another form: 'An unknown square is the same as a given rectangle on the unknown square-side and a given area', exemplified by a problem. Ea-shar-ili elects to see a picture demonstration first, then the procedure is deduced, and the solution to the problem is found. The Head Scribe demonstrates another way for picturing such problems. He commends Ea-shar-ili for grasping the meaning of the mathematics and for his concern for justice. He forecasts a great future for him, and celebrates the scribes' role in seeking justice by reading an old praise-poem.
Sources

The problem is Problem 2, BM 13901; method given in Katz & Parshall (2014), *Taming the Unknown*, 25-26]; statement and diagram given as Problem (ii) in Katz (2007), *The Mathematics of Egypt, Mesopotamia, China, India and Islam*, 102-104. The Head Scribe's commendation is paraphrased from a cuneiform dialogue in Robson & Stedall (2010), *The Oxford Handbook of the History of Mathematics*, 217-218 (paperback edition), apparently between an advanced student and a younger student. Robson sees the interchange as illustrating the importance of accurate land surveys for legal reasons, e.g. inheritance, sales, and harvest contracts. The praise poem is paraphrased from Katz (2007), 91-92, and (in different translation) Robson & Stedall (2010), 218.

Christopher Hollings: *Thomas Eric Peet, historian of mathematics*

In 1923, the English Egyptologist Thomas Eric Peet (1882–1934) published an edition of the Rhind Mathematical Papyrus — one of the main sources, held in the British Museum (P. BM EA 10057-8), on ancient Egyptian mathematics. Although a facsimile of the papyrus had been published at the end of the nineteenth century, and certain aspects of it had been studied by other Egyptologists, Peet's edition, translation and study was the first comprehensive treatment of its mathematical content. In his commentary on the papyrus, as well as in the small number of other works that he published on ancient Egyptian mathematics, Peet displayed a sensitivity to historical context that was not present in the works of most other historians of ancient mathematics during the 1920s and 1930s. Perhaps for this reason, Peet's work on ancient mathematics (his edition of the Rhind papyrus aside) appears to be little known beyond Egyptological circles. In this talk, I will describe the content of Peet's study of ancient Egyptian mathematics, and consider his approach to the subject in comparison to that of contemporaneous historians of mathematics.

David Horowitz: *Mathematics and Morality: A very early manuscript of Colin MacLaurin*

Colin MacLaurin (1698-1746) was one of the preeminent Scottish mathematicians of the eighteenth century. He is best known for his work on gravitation, geometry and fluxions. However in 1714 at the age of 16 he withdrew from Divinity School at Glasgow University and returned to Kilfinan (Argyllshire) to live with his uncle who was the minister at the local parish church. There MacLaurin wrote a short mathematical treatise dealing with morality. De Viribus Mentium Bonipetis ("On the good-seeking forces of the mind") is a youthful attempt to combine calculus, Newtonianism and religious tenets of the Church of Scotland. The Latin manuscript seems to have remained completely unknown for over 250 years until rediscovered in the late twentieth century.

Only a handful of people have examined the 10-page handwritten text which currently resides in the archives of the University of Edinburgh Library. De Viribus has never been published nor otherwise made generally available. Erik Sageng may have been one of the first to unearth it while doing research for his 1989 doctoral dissertation on MacLaurin (Princeton University). In 2008 Ian Tweddle (University of Strathclyde) wrote a careful translation of the manuscript along with detailed notes on the mathematics contained therein. Subsequently Judith Grabiner (Pitzer College) and Olivier Bruneau (Université Paris-Saclay) offered additional insights into De Viribus. However all of these dealt exclusively with the mathematical content of the manuscript and not with the moral or religious ideas which were the motivation for MacLaurin's treatise.

This seminar will try to make sense of the mathematical morality in De Viribus from the perspective of early eighteenth-century Scotland. Throughout his life MacLaurin was a deeply religious man. In De Viribus he connects his terse and often-cryptic Newtonian-style statements with theological concepts such as original sin and The Tempter. It will be explained how deciphering these connections offers a unique insight into how MacLaurin (and his scientific contemporaries) approached the Scottish Enlightenment and a Church of Scotland undergoing profound doctrinal changes. De Viribus can also be viewed as an early attempt to apply a nascent Newtonianism to a non-scientific topic. Newton's Principia had been written over twenty-five years earlier, but MacLaurin was one of only a few people in England who fully understood it at the time. This session will try to make intellectual and historical sense of MacLaurin's technical methods as well as his moral attitudes and principles. The interface between these provides a glimpse into MacLaurin's thought processes, mathematical motivations and societal feelings about early Scottish presbyterianism.

Cynthia Huffman: *Agnesi vs. Colson: Did Place Matter?*

Maria Gaetana Agnesi, born into a wealthy family in Italy, demonstrated remarkable intellectual talent from a young age. She is best known for her two-volume work on algebra and calculus, Instituzioni Analitiche ad uso della gioventù italiana (Foundations of Analysis for the Use of Italian Youth), published in 1748 in Milan. The book was written in Italian as a teaching text and received high praise from a committee of the Académie des Sciences in Paris for uniformly synthesizing and clarifying the work of others on calculus. According to the Dictionary of Scientific Biography, "this book won immediate acclaim in academic circles all over Europe and brought recognition as a mathematician to Agnesi." In fact, it was used as a standard text in Europe for over 100 years.

The English mathematician Reverend John Colson supposedly learned Italian so that he could translate Agnesi's text into English. Colson had previously translated some of Sir Isaac Newton's work into English. Today many people know of Agnesi due to Colson translating the name of a cubic curve she discussed in Instituzioni Analitiche as "the Witch," resulting in the curve becoming commonly known as the Witch of Agnesi.

Agnesi and Colson were writing during a time not long after the heated Newton-Leibniz calculus controversy. As an Italian, Agnesi shared living on the European Continent with the German Gottfried Leibniz. Colson, however, was English, as was Sir Isaac Newton. In this presentation, we will compare specific examples from Maria Agnesi's Instituzioni Analitiche with the corresponding text of John Colson's English translation Analytical Institutions to investigate the question of whether or not place made a difference.

Manuel Madrano: *As if their knots were letters': Uncovering Analogy in Colonial Accounts of Andean Khipu Mathematics*

Following the Spanish conquest of the Andes in 1532 AD, the colonial chroniclers endeavored to write a history of the Inka Empire that described khipus—the knotted-string devices that the Inkas used, in lieu of graphical writing, to record numerical and narrative data. The flexibility of khipu recording is evident in the confusion of their colonial descriptions. Numerical khipus stored the results of mathematical calculations that the Inkas performed to administer their empire—an accountancy so precise that, according to León (1553), "not even one pair of sandals would be lost." Chronicles describing numerical khipus often state that their information storage capacity was limited to mathematical data. However, other early-colonial writings describe a class of khipus that recorded narrative information, utilizing numbers as linguistic labels to record genealogies, myths and historical accounts. Others still describe fully phonetic khipus, which could be "read" by cord specialists who employed widely conventionalized interpretive techniques. The incongruity of the colonial accounts is a consequence of their central disagreement—the extent to which the base-10 numerical values that the Inkas recorded with knots on their khipus were narrated as such; or, whether these values might also comprise a mapping to extra-numerical information. In developing their respective positions, the chroniclers faced a daunting task: describing three-dimensional khipu recording to European audiences accustomed to two-dimensional graphical writing. By and large, the explanatory strategy chosen by the chroniclers was analogy—comparing khipus to books, syllabi or abacuses; their knots to rosary beads or (per Vega [1609]) alphabetic letters; and their readers to accountants, professors or authors. Khipu analogies are ubiquitous in the colonial accounts, which confronted sixteenth-century European audiences with a multitude of descriptive permutations. The paper synthesizes these descriptive analogies to situate the conceptual location of khipu mathematics in the early-colonial European imagination. More than calculation storage devices, khipus recorded a relational number system—its values related by visual-tactile associations intimately associated with Andean lifeways. I argue that close reading of the khipu analogies in colonial chronicles forms a novel inroad to identifying the mathematical confusions of early-colonial contact. Further, I find that many lingering misconceptions in modern khipu decipherment efforts can be traced to explanatory confusions deposited in these earliest colonial analogies. Paradigmatic ways of analogizing khipus were introduced to the sixteenth-century Iberian public, which, ever recirculated, cemented dominant ways of "knowing" Andean numeracy. Targeted historiography offers a powerful means of distilling native Andean mathematics—

recorded in the primary sources of the Inkas—from the conflicting accounts of the colonial chroniclers.

Athenasia Megremi: *Mathematical commentary as a mathematical milieu: being introduced in the world of the Palatine Anthology (10th century CE)*

Arithmetical place and practice are the subject of this presentation. How do the scholia found at the margins of book fourteen (IV) of the Palatine Anthology provide us with such valuable information, is of course inferred. The study of a text like the codex Palatinus is a case of unraveling a story of what the edition of an anthology ca. the 10th century CE means. The compilation of an anthology of epigrams by K. Kefalas (9th or 10th century CE) was remade and reedited less than a century later in a scriptorium where the marginalia produced give us enough information to understand how the compilation process and the work of scribes had evolved. The story told by the scholia on the arithmetical epigrammes of the Palatine Anthology all attest to the mathematical culture of the editors in charge of the compilation. Offering solutions to problems and preserving a series of solutions regarding one single problem are two aspects that concern not only the evolution and transmission of commentary tradition but also of the people preserving mathematical traditions, creating a sort of implicit historical narrative.

In this presentation, part of a work in progress about the importance of the Book IV of the Palatine Anthology for the history of mathematics, we will track the process behind the compilation of the codex, the work of the scribes and editor, the mathematical content and the practice of Arithmetic in the ca.10th century.

Duncan Melville: *The mathematical papers of RFA Lee*

In the early 1800s, Rachel Frances Antonina Lee (1774—1829) spent a considerable amount of time and effort in the preparation of a “Course of Mathematics”. Although the work went through three partial drafts covering hundreds of pages over a period of several years, it was never completed and never published. Its survival, and that of her other mathematical papers, is due to accidental and fortuitous circumstances.

We will explain the background and life of the author and discuss the content, both mathematical and philosophical, of the “Course of Mathematics”. Given the absence of any comment on her mathematical education or attainments in other biographical sources, the survival of such subterranean mathematics suggest historians should be cautious in our assessment of the range and depth of female mathematical knowledge and interests in the early 19th century.

Amirouche Moktefi: *Hugh MacColl, mathematical reviewer for* The Athenæum

Scottish mathematician Hugh MacColl (1837-1909) moved to Boulogne-sur-Mer (France) in 1865. He lived there as a schoolmaster for the rest of his life and produced numerous works in mathematics, philosophy and fiction. His work in logic has recently been re-discovered and is said to have opened the way to logical pluralism. MacColl’s position as an outsider living at the periphery of Britain may have contributed to the originality of his work. Indeed, he lived within an important British community abroad, including a small group of mathematicians who regularly contributed to the *Educational Times*’ mathematical columns. He knew of the main mathematical works published by his British contemporaries. Yet, due to his life abroad and his failure to secure an academic position, he was not part of the British intellectual circles and felt no commitment to any school or tradition. This is evident in logic where his work is free of the Boolean equational canon. It is thus unsurprising to see his British colleagues, notably W. S. Jevons and J. Venn, strongly oppose to him. Interestingly, MacColl's ideas were better appreciated in France, Germany and the United States.

We consider MacColl's peculiar position as an outsider and his relation to the mathematics of his time through his reviews to the *Athenaeum*. Indeed, MacColl acted as the main reviewer of mathematical books for this popular Victorian journal. The choice of MacColl, schoolmaster at Boulogne-sur-Mer, for such a position is itself a remarkable mystery. Indeed, it must be reminded that such a task was previously undertaken by Augustus De Morgan, Professor of Mathematics at London University (Despeaux and Rice 2016). These reviews were anonymous and have long remained unknown (Abeles and Moktefi 2011). However, the journal editor's marked copies, which reveal the identity of the reviewers, have survived. A survey shows that between 1886 and 1907, MacColl reviewed about 250 books in mathematics, logic and various related subjects. This material offers an original standpoint to observe MacColl's familiarity and appreciation of the British mathematics and mathematicians of his time. In this research, we survey MacColl reviews and describe their scope. We particularly consider how these reviews mirror MacColl's search for recognition through the references his made in those anonymous reviews to his own works.

F. F. Abeles and A. Moktefi (2011), "Hugh MacColl and Lewis Carroll: crosscurrents in geometry and logic", *Philosophia Scientiae*, 15 (1), 55-76

S. E. Despeaux and A. C. Rice (2016), "Augustus De Morgan's anonymous reviews for 'The Athenæum': A mirror of a Victorian mathematician", *Historia Mathematica*, 43 (2), 148-171

Rogério Monteiro: *Circulation of Auguste Comte's mathematical texts between France, Brazil and Chile at the end of Nineteenth Century*

In the edition of December 1896, the French periodical "Revue des deux mondes" published a long paper by the French mathematician Joseph Bertrand on Auguste Comte and his relationship with the French Polytechnical mathematicians. Beyond a tumultuous history, including his expulsion of the Polytechnical School of Paris and a marriage with a prostitute, Comte is presented as a scientific farse by Bertrand. Errors of mathematics and physics in Comte's works are pointed out by the French mathematician in order to dismantle Comte and the positivism as an alternative way for traditional religions. One month later, members of the Brazilian positivist church with the collaboration of French and Chilean positivists started a coordinated reaction against to Bertrand. On both sides of the Atlantic, they published articles in newspapers and small brochures in both Portuguese and French. In my talk, I would like to present the main arguments of the episode, providing also an analysis of the transatlantic network of printed matters and engineers which provides the material conditions of mathematical theories circulation between South America and Europe at the end of Nineteenth Century.

Madeline Muntersbjorn: *Feature Language and the History of Algebra*

To the novice, the most salient feature of algebra is its abundance of symbols even though early algebraic texts do not exhibit much formal notation. Algebra's earliest practitioners were mathematicians who used ordinary language in extraordinary ways to represent problems and their solutions with unprecedented precision. The translation of these story problems into formal symbols happened but centuries later; how many is subject to debate.

Robert Moses, known for teaching algebra as a social justice project, breaks the process of learning algebra down into five steps. First, students share an experience that is later represented by individuals; these individual representations are then shared using “people talk” before being translated into “feature talk.” Finally, feature talk is translated into formal symbols. Moses’ emphasis on feature talk, as an intermediary between ordinary language and mathematical notations, is inspired by Quine’s observation that scientific discourse involves “the regimentation of ordinary language.” Specifically, this regimentation involves the transformation of what we say in natural languages into more logically perspicuous linguistic expressions, more amenable to formal symbolization and algorithmic manipulation than their ordinary counterparts. Historians of algebra and philosophers of mathematical practices would do well to more precisely mark the differences between transitions four and five, from informal talk about mathematics to precise mathematical talk, on the one hand, and from precise mathematical talk to exact symbolic expressions, on the other. Too often these transitions are elided, especially when historical content is translated into modern notation sans caveat. By paying closer attention to the differences between these transitions, scholars will be better able to disambiguate patterns of mathematical expression (e.g., abbreviation, substitution, standardization, etc.). One of the most important morals from the history of early modern algebra is that becoming fluent in “feature language” was a necessary precondition for being able to solve an increasing number of problems. Historical narratives that shed light on this fluency should be relevant to those educators seeking to help students go from talking casually about story problems to speaking the languages of mathematics more correctly.

Maurice OReilly: *Developing an online exhibition of selected mathematical works from Marsh’s Library, Dublin: an early modern library through the eyes of undergraduate mathematics students*

In spring 2019, six students taking mathematics on a joint honours BA programme at Dublin City University took part in a project exploring some of the mathematical works in Marsh’s Library, Dublin (incorporated in 1707). At the Maynooth Conference on the History of Mathematics (BSHM/IHoM5, August 2019), the author presented an initial report on this project, outlining its context and how each student engaged with two works, focussing on the author, the content and the context of each work.

Of the twelve 17th-century works chosen, six were published in the 1670s and 1680s (Huygens, Barrow’s Archimedes, Hooke, Petty, Baker and Norwood), four predated this period (Galileo, Viète, Oughtred and Descartes) while two appeared in the last decade of the century (Dechales and L’Hospital). These twelve cover a broad range of both classical and novel mathematics known in the period, including fundamental perspectives (e.g. Galileo and Descartes), ‘new’ applications (e.g. Huygens and Petty) and textbooks (e.g. Oughtred and L’Hospital). Although they comprise less than a tenth of the Marsh’s mathematical collection, the breadth of the twelve works can be considered representative of that collection.

In this presentation the emphasis will be on the final stages of preparation for the online exhibition and a retrospective reflection on the accomplishments and challenges of the project. The manner in which the students addressed the multiple challenges of unfamiliar original sources, notation and, often, languages reveals new opportunities for working with early modern material.

Daniel Otero: *From latitudo formarum to graphs of functions*

Nicole Oresme (1323-1382), the medieval Norman cleric and scholar, is well known for having developed a geometrical method for the study of kinematics, which he termed latitudo formarum, "latitude of forms," and which he employed some time in the 1350s to produce a proof of the Mean Speed Law for falling bodies. Today, his graphical method is generally cited as a precursor to the modern method of graphical representation of a function $f(x)$ of a real variable, the locus of points $(x, f(x))$ in a coordinate plane. But to what degree can we claim that Oresme prefigured the use of graphs to study the behavior of functions in modern analytic geometry? This paper will attempt to elucidate some of the historical and conceptual connections between Oresme's latitudo formarum and modern analytic geometry to shed light on the evolution of these geometric methods in mathematical analysis.

Raffaele Pisano: *Reading the Culture of Machines Bordering Early Mechanics and Mathematical Practices: Influential and Heritages within the Relationship Physics–Mathematics into History*

In the western countries the conceptualisation of the equilibrium within the structural–resistance mechanics is usually associated to Galileo's *Discorsi e dimostrazioni matematiche sopra a due nuove scienze* (1638)—in those portions of the text often ignored in favour of his more famous kinematics of falling bodies and projectile motion, presented in the second part of the treatise. However, there is, as it were, often–unknown cultural tradition, based in elements of Arabic science and Medieval Latin manuscripts/editions that can help us to understand the particular strands of theory and practice that played into Galileo's work on the strength of materials.

15th–16th centuries. The study of machines/mechanics referred to the Science of weights (Scientia de ponderibus) which itself displayed two main types of scientific–cultural approaches: one related to Mechanical Problems, where the equilibrium of a body stemmed from two contrasting tendencies; while the other more specifically mathematical approach derived from Archimedean works: the equilibrium reduced to the evaluation of the centre of gravity of a body. It is possible to show an interesting and complex interaction amongst Culture of machines & Science of weights, considerations of the characteristics of fortifications and certain new initiatives in early Mechanics. The works on fortifications (not only military treatises) played into the existing interplay of the two traditions to give birth to new analyses of equilibrium much earlier than Galileo's mechanics.

This talk will illustrate these points by explicating recent works of mine (and others within a specialist literature) which have analysed in Culture of Machines, Practical mathematics—without being Practice and bordering early Mechanics—and—Scientia de ponderibus. This analysis also brings the history of Renaissance Mechanics into even closer relation than previously realized with the more practical and applied areas of field, especially the burgeoning domain of Fortifications Treatises. For sake of brevity, this paper will present the historical weight of Culture of Machines (reasons, conflicts and values) with respect to Scientia de ponderibus (i.e., by Tartaglia, Galileo and Beekman) and related manuscripts on fortifications (Tartaglia, Galileo and Lorini)

Madeline Polhill: *Bryn Mawr College, 1885 - 1940: The People, Places and Practices that Helped One Institution Thrive*

With the 100th anniversary of the passage of universal suffrage for many American women having arrived in late summer 2020, and as women continue working to break through glass ceilings in academia, government, and industry, looking to the past helps provide inspiration for members of today's mathematical community to support women students and colleagues. Though its yearly total enrollment never surpassed 640 students in its first 55 years of existence, Bryn Mawr College awarded more mathematics PhDs to women from 1885–1940 than all but two other American institutions. Together, administrators M. Carey Thomas and Marion Edwards Park and mathematicians Charlotte Angas Scott and Anna Pell Wheeler, in their efforts to support women, advanced mathematics and the women around them.

An exploration of the existing scholarly body and archival work at Bryn Mawr College and the Library of Congress reveal several common, timeless ways in which these four women helped create a haven for mathematics at Bryn Mawr. Perhaps not surprisingly, the linking threads between these women's stories relate to the core ideas of *people*, *places*, and *practices*. By committing themselves to the *practice* of maintaining the highest academic standards, these leaders effectively proved the capability of women to succeed in mathematics and established a bastion of absolute mathematical excellence. Recognizing the importance of physical *places* within scholarly life, these women provided Bryn Mawr's students and faculty members with dignified and well-designed libraries, work areas, and private spaces that supported high-quality scholarly efforts. Engaging in an intentional *practice* of creating connections with other *people*, institutions, and societies in the mathematical community, these four figures helped introduce Bryn Mawr's mathematics department to mathematical research they may not have encountered otherwise and provided them with access to enhanced scholarly materials, course offerings, and professional opportunities. Throughout the twentieth century, some of these women supported the *practice* of reducing discrimination by opening the college to a larger group of scholars. In turn, these leaders recommitted the college to academic excellence, allowed for vibrant mathematical interactions, and helped bring the greatest female mathematician of the twentieth century to Bryn Mawr. Finally, these women's own biographies reveal their roles as loyal and motivating mentors. As pioneers in academia in their own rights, these women proved to their students and colleagues the possibilities available to devoted women in mathematics. Their inspirational teaching inspired the next generation of women in mathematics, and their unwavering encouragement provided necessary care for their students' personal needs.

This talk explores these women, their legacies, and the inspirational template for success they offer as today's mathematical community aims to champion women and all underrepresented groups in mathematics.

Chris Pritchard: *Life of pie: William Playfair and the advent of statistical diagrams*

The bar chart, pie chart and time-series chart were conceived at the end of the eighteenth century. This is the story of how a scoundrel from a distinguished Scottish family helped to create a visual mathematics with a focus on impression rather than rigour, a mathematics that was viewed sceptically by specialists but ultimately helped non-mathematicians to understand complex issues.

Robert Rogers: *Using the Story of Calculus to Teach Differential Calculus*

The typical differential calculus course starts with foundational topics such as limits and continuity, even though historically, these were among the last topics developed. This talk proposes that it is pedagogically advantageous to follow history. Students can review their precalculus skills by examining ad-hoc techniques for optimization and tangents before learning the "New Method for Maxima and Minima, as Well as Tangents" of Leibniz and Newton. Once the rules for this "differential calculus" are learned, students can immediately utilize this new tool on problems that are not easily solved by the aforementioned ad-hoc techniques. After students realize the power of this calculus, we come back and consider some of the subtleties. At this point, students will be in a better position to appreciate these foundational aspects. The author will include materials from an open source calculus book he is co-authoring which utilizes this approach.

Dirk Schlimm: *MacColl's reflections on logical notations*

The Scottish mathematician and logician Hugh MacColl (1831-1909) introduced several innovations in symbolic logic and pioneered the study of modal and other logics. In this talk, I will focus on his contributions to the development of propositional logic and his reflections on general principles for the design of notations.

Let me briefly illustrate some of MacColl's original contributions. In MacColl (1877), he famously declared that his logical variables denote statements, instead of classes. This allowed him to abandon the equational form of logical statements that was introduced by Boole. Moreover, he also introduced a logical symbol ':' for implication independently of Peirce and Frege. These changes had a dramatic effect on logical no- tation, because, with '=' now standing for the equivalence of two statements, MacColl could use multiple equation signs in a single formula, such as $(a = b) = (a : b)(b : a)$ (MacColl 1878, 16).

However, MacColl did not just employ new conceptions and notations, but he also discussed his views on the design of notations throughout his series of papers on the'calculus of equivalent statements' (6 papers, 1877–1896) and on 'symbolic reasoning' (8 papers, 1880–1906). In my talk, I will present and discuss MacColl's principles for good notations, also in light of later criticisms by Ladd (1883, 24–25) and Schroder (1881, 94).

Anna Kiel Steensen: *The mathematical use of graphic position in C. F. Hindenburg's combinatorial school*

How can we use mathematical texts to describe practices that use diagrams and diagrammatic features of written language? What is the relation between how a reader interprets the diagram and how actors consider the mathematical status and function of the diagram? In this talk, I will address these questions in the case of German mathematician C. F. Hindenburg (1741 – 1808).

Specifically, I focus on how Hindenburg (e.g. [1795]) makes mathematically significant the relative positions of individual letters, numbers and line segments (not geometric position, but graphic position as a spatial feature of letters etc.). Following Knuth, who wrote that Hindenburg gave "combinatorial significance to the digits of numbers written in decimal notation" [2006: 69], I am interested in a specific semiotic process: how interpretations of diagrams arise from the interplay between text and diagrams, and how the interpretations relate to Hindenburg's mathematical use of position.

To describe this process, I apply a structural-analytical approach, which does not presuppose that the interpretation of the diagram is given or universal, but constructs it in the analysis.

The heritage of Hindenburg's combinatorial school is generally regarded as limited when it comes to defining new mathematical concepts and proving theorems. The present project opens the question of the school's influence: its semiotic work may have contributed to opening up a new domain for mathematical consideration. The project thus indicates how a local visual-textual practice can influence wider mathematical practice.

James Tattersall and Shawnee McMurran: *A Cambridge Correspondence Course in Arithmetic for Women*

In late nineteenth century Victorian England, when interest in the education of women began to gain momentum, an association of dedicated women and fellows at Cambridge colleges began coordinating a series of lectures for women. Geared toward women with interest in the teaching profession, the lectures were structured to provide preparation for the Cambridge Higher Local Examination. Examiners awarded certificates to successful candidates that would qualify them for a teaching position. Particularly prized was a certificate denoting honors. Lectures were offered each term, each with a focus on subjects relevant to the exam. Obstacles included the procurement of lecturers, as well as appropriate accommodation for the women who ventured to Cambridge in order to attend the lectures. Lecturers included mathematicians Arthur Cayley, William Kingdon Clifford, and Norman Macleod Ferrers, the astronomer John Couch Adams, the economist Alfred Marshall, and the logician John Venn. In 1873, to ensure more support for the venture, the Lecturers' Association merged with the Association for the Promotion of Higher Education for Women. In this presentation we focus on the first ten years of mathematical lectures and discuss the 'Lectures for Women' organization that formed the basis for the foundation of Newnham College, Cambridge. We include mathematical questions from early examinations.

Valerie Lynn Therrien: *On Counting as Mathematical Progress: Kuratowski-Zorn's Lemma and the Path Not Taken*

In her *Naturalism in Mathematics*, Maddy claims that historical case studies give us sufficient reason to exclude extra-mathematical considerations from our account of mathematical progress. Indeed, she vouches that historical case studies can be tested against the predictions of a reconstructed means-end analysis. In this paper, we will take up this formidable challenge. We aim to do so *via* a carefully chosen case study designed to test the limits of a rational reconstruction's ability to predict not only the path taken by mathematics, but also the path *not* taken by mathematics: the case of the Kuratowski-Zorn Lemma. Can Maddy's framework account for why Zorn's Lemma counts as mathematical progress, but Kuratowski's prior equivalent maximal and minimal principle does not? While Maddy has done ground-breaking work in rationally reconstructing the path taken by set theory, it is not clear that her account can pro- vide a convincing rationale for the path *not* taken. Our conclusion is that, while Maddy's account provides a razor-thin margin of success, it also does not take into account salient extra-mathematical considerations. Ultimately, it is unlikely to be convincing to anyone not epistemologically committed to mathematical naturalism.

Anne van Weerden: *Sir William Rowan Hamilton: "a studious and happy life"*

There are many biographies of varying lengths of Sir WR Hamilton, in print and online, which present negative views of his private life, describing it as having been difficult and very unhappy. It is also frequently asserted that as a result, he sought refuge in alcohol. At the fifth Irish History of Mathematics Conference at Maynooth in 2019, I presented evidence to show that Hamilton was not alcoholic, and discussed where this alcoholic reputation originated from. In this presentation I will, using primary and secondary sources, show that he also was not unhappy. Although Hamilton had his times of grief, his life which revolved around his marriage, his family and his mathematics, was, as he called it, "a studious and a happy one".

Lukas M. Verburgt: *"Any scheme of allowing for errors is a makeshift for removing them": Francis Galton, John Venn and the unofficial Cambridge Anthropometric Laboratory*

The story of statistics before 1900 is one of a logic common to every science that emerged from the interplay of two developments: the combination of observations and the use of probability mathematics. Both having separate beginnings, these two developments intersected in the first decades of the 19th century, only to spread as a single method horizontally – across scientific disciplines – and vertically – in terms of technical sophistication.

This grand story comes at the price of a loss of historical accuracy. The main reason is that its focus on abstract concepts neglects both the people, places and practices involved in the creation of sufficiently abstract statistical knowledge. The present paper aims to illustrate that the history of statistics can be told differently through a focus on the human, local and material aspects mathematics: and, *vice versa*, it gives an example of what appears into view when adopting this alternative narration.

Drawing on fresh archival research, I will describe a fascinating, yet little-known episode in the history of statistics: Francis Galton's collaboration with John Venn in an unofficial anthropometric laboratory at Cambridge between 1887 and 1889. My focus will be on the various difficulties Galton and Venn experienced in their joint endeavor, which ranged from finding a suitable room and weighing the reliability of
instruments to aligning statistical techniques with measurement results. In doing so, Galton and Venn were forced to use their polymathic skills and Cambridge contacts to come up with hands-on ways of finding out which statistical errors were theoretically relevant and which were due to practical mistakes.

The paper concludes by placing the Galton–Venn laboratory into the context of the emergence of psychometrics at Cambridge and by considering the wider importance of its (unsuccessful) results for the discipline of statistics in the 1880s-1890s and beyond.

David Waszek: *Notational naturality in Leibniz and Lagrange*

As Jamie Tappenden (2008a,b) has noted, contemporary mathematicians frequently discuss what the 'right' (or 'natural') definitions or concepts for a given area are. This presupposes a subtle attitude to definitions. On the one hand, they are assessed on the basis of their ability to streamline current knowledge: for in- stance, a definition may be chosen because it allows stating general theorems, or writing down important proofs, in a (comparatively) simple way. On the other hand, definitions adopted on such grounds are often judged well-adapted to the 'nature of the subject', and are therefore used to guide further research.

One might think that such a methodological stance is typical of what historians sometimes call 'modern' mathematics (e.g., Benis-Sinaceur 2002; Gray 2008) or at any rate that it is no older than the nineteenth century. This paper, however, ar- gues that a strikingly similar attitude can already be found in the late 17th century in Leibniz's work, and then traced into the 18th century, but in a form that con- cerns notations rather than definitions or concepts. Indeed, Leibniz, whose peculiar notational practices have already been noticed (e.g., Serfati 2008; Knobloch 2016), tended to adopt notations just because they allowed writing simple and general formulas, and in turn, to treat formulas that were simple and general (in his nota- tions) as plausible – in a sort of reasoning that sometimes seems almost circular.

I shall focus on a clear case of such judgements of 'notational naturality', namely a particular notation introduced by Leibniz in 1694: the exponential notation for differentials (i.e., '$d3x$' instead of '$dddx$', and so on). I shall review why Leibniz adopted it and how it shaped his, and Johann Bernoulli's, research and eventual discoveries. I shall then follow it into 18th century France, where it played an im- portant role in the emergence of the so-called 'calculus of operations'. This phrase refers to a corpus of works by, among others, Lagrange, Laplace, Arbogast, and Servois, in which operators, for instance the 'd' of differentiation, are treated as if they were algebraic quantities whose 'powers' are interpreted as iterated applica- tions of the operator (Koppelman 1971; Lubet 2010). My goal will be to explore the attitudes of these various authors, in particular Lagrange, towards the notation: to what extent and in what sense did they interpret its ability to summarise known results and yield novel ones as a sign of what we might call 'naturality'? How did they relate this 'naturality' to the possible existence of new mathematical objects, namely operators?

Diana White: *The role of faculty development in supporting adoption of curricular modules to teach undergraduate (tertiary) mathematics using primary historical sources*

There are a variety of evidence-based teaching practices that are known to improve student learning in mathematics and other STEM disciplines. While faculty uptake of these practices remains a barrier to student success, faculty development workshops are one approach to support faculty in incorporating new teaching techniques. In particular, workshops increase the adoption of inquiry-based learning techniques by faculty, and these approaches tend to be favored by students. Workshops are widely seen as an effective professional development strategy, and in fact Khatri and colleagues (2013) noted that the most effective means of spreading educational innovations is "multi-day, immersive experiences with follow-up interaction with the [workshop facilitators] as participants implement the new strategy".

Laursen and colleagues (2012, 2014, 2016, 2018) have studied the impact of professional development workshops for supporting mathematics faculty in implementing inquiry-based learning, but to our knowledge, no studies exist related to effectiveness of workshops to support faculty in incorporating primary sources into their teaching of undergraduate (tertiary) mathematics.

We report on findings from two workshops, each three-days in length, to support faculty in the use of Primary Source Projects (PSPs). PSPs are inquiry-based curricular modules designed to teach undergraduate mathematics from primary historical sources rather than standard textbooks. Each PSP is designed to cover its topic in about the same number of course days as would be needed from a more standard textbook-based approach. We briefly report results on the effectiveness of the workshops on participant's (n=78) learning and preparedness to implement PSPs (changes from pre- to post-workshop as well as one-year out), as well as their overall reactions to the quality of the workshop. We present both qualitative and quantitative results related to their perceived impact of PSPs on themselves and their students as well as challenges they may have faced during implementation.

Finally, we present results from semi-structured follow-up interviews that were designed to assess the longer-term impacts of PSP implementation, including experiences implementing more than one PSP, and themes related to the challenges and obstacles faced during implementation.

Maria Zack: *Early Computations on the Cycloid*

Many well-known mathematicians of the seventeenth and eighteenth centuries studied the cycloid. These include Galileo, Roberval, Descartes, Pascal, Wallis, Huygens, Fermat, Newton and Leibniz and more than one Bernoulli. This talk will consider the quadrature computations of Galileo's student Evangelista Torricelli. The computations were completed sometime before April 1643 when Calvalieri sent a letter to Torricelli congratulating him on the findings. Torricelli's results were published in an appendix in his *Opera Geometrica* (1644).