Maths is good for you!

History of mathematics for young mathematicians

Platonic Solids

Mathematics Master Class

Name

Date

Teacher

Platonic and Archimedean Solids

There are only five regular convex polyhedra that can exist in our three dimensional Euclidean space. These polyhedra were known since the ancient times. Plato (427-348 BC) wrote about them in his work Timaeus (written in about 340 BC) to identify five principles upon which everything is modelled: he identified these principles as fire, earth, air, water, and cosmos (or divine force). He identified each with the elementary principle

Tetrahedron – fire
Cube – earth
Octahedron – air
Icosahedron – water
Dodecahedron – cosmos or divine force

Euclid's *Elements* (c. 300 BC) list all these solids in mathematical way, giving their properties in the last book, book XIII.

Around the same time Archimedes was working on Platonic solids and discovered new polyhedra which are now called Archimedean polyhedra. The difference between platonic and Archimedean polyhedra is that, while the former are regular polyhedra containing only regular and same faces, the latter contain two or more different types of faces (which are also regular polygons).

Have a look below and at the next page where all the Platonic and Archimedean polyhedra are listed.

Platonic Solids

cube

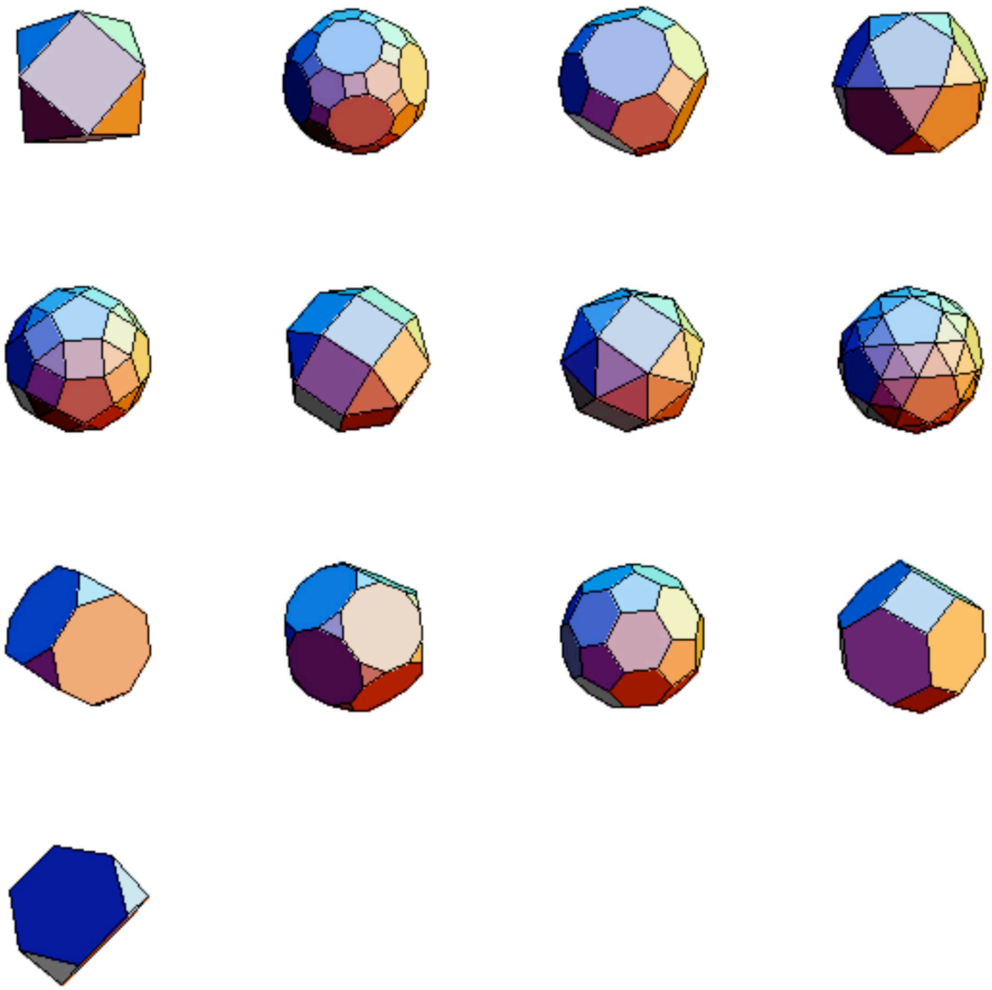
dodecahedron

icosahedron

octahedron

tetrahedron

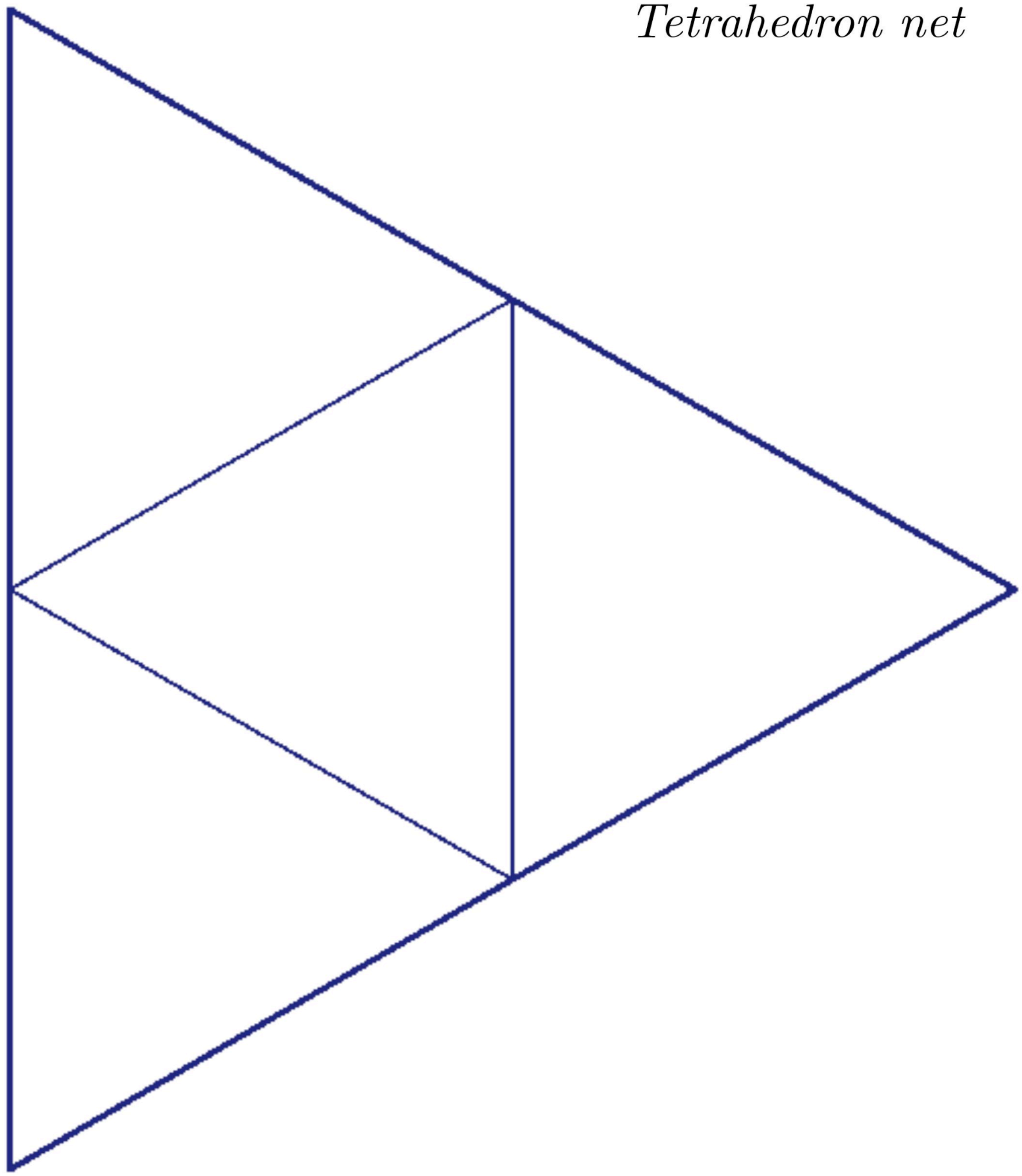
Archimedean Polyhedra

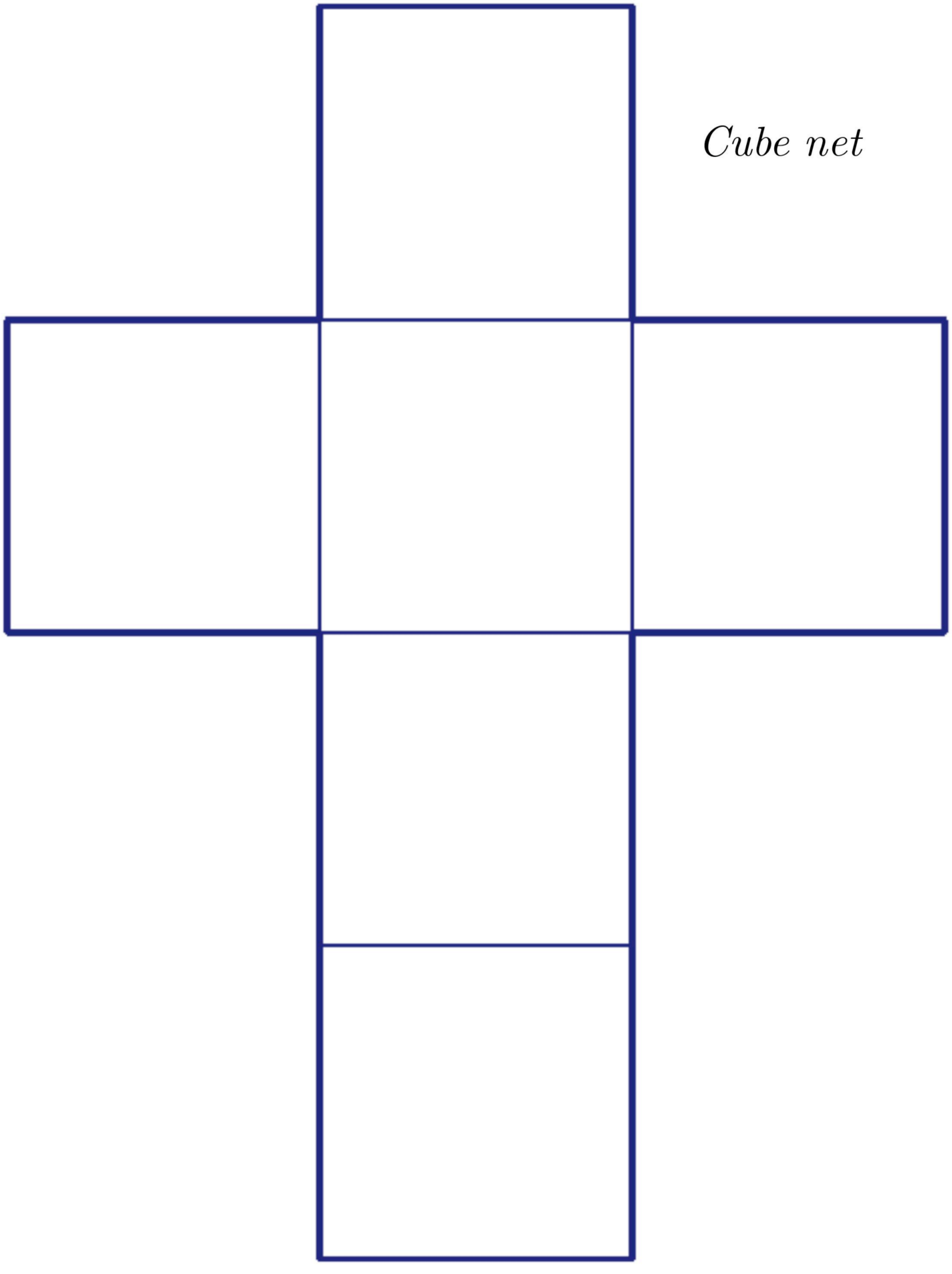


Their names are
cuboctahedron
great rhombicosidodecahedron
great rhombicuboctahedron
icosadodecahedron
small rhombicosidodecahedron
small rhombicuboctahedron
snub cube
snub dodecahedron
truncated cube
truncated dodecahedron
truncated icosahedron
truncated octahedron
truncated tetrahedron

For the following exercise you need to cut out the nets and make Platonic solids out of them.

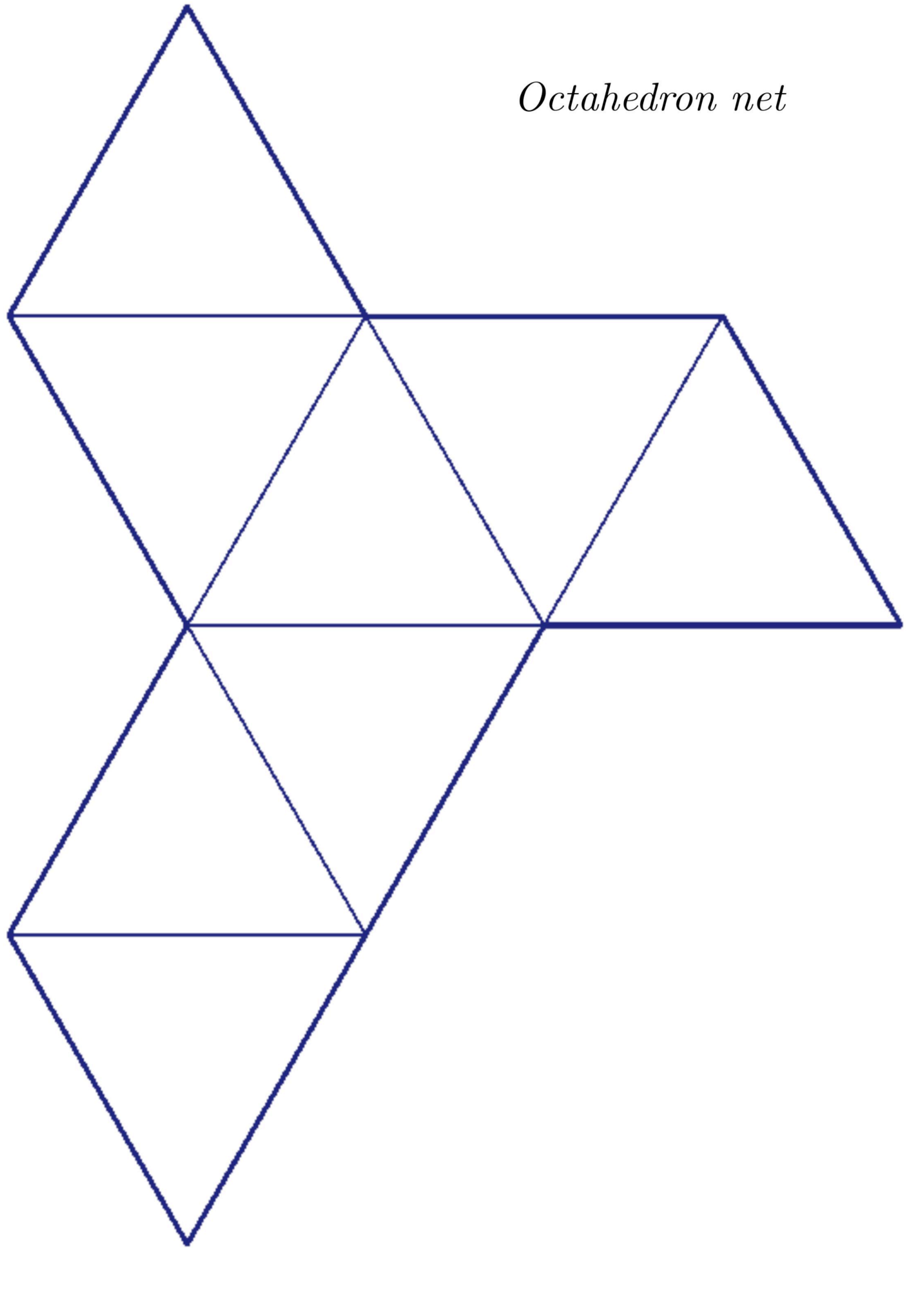
Tetrahedron net



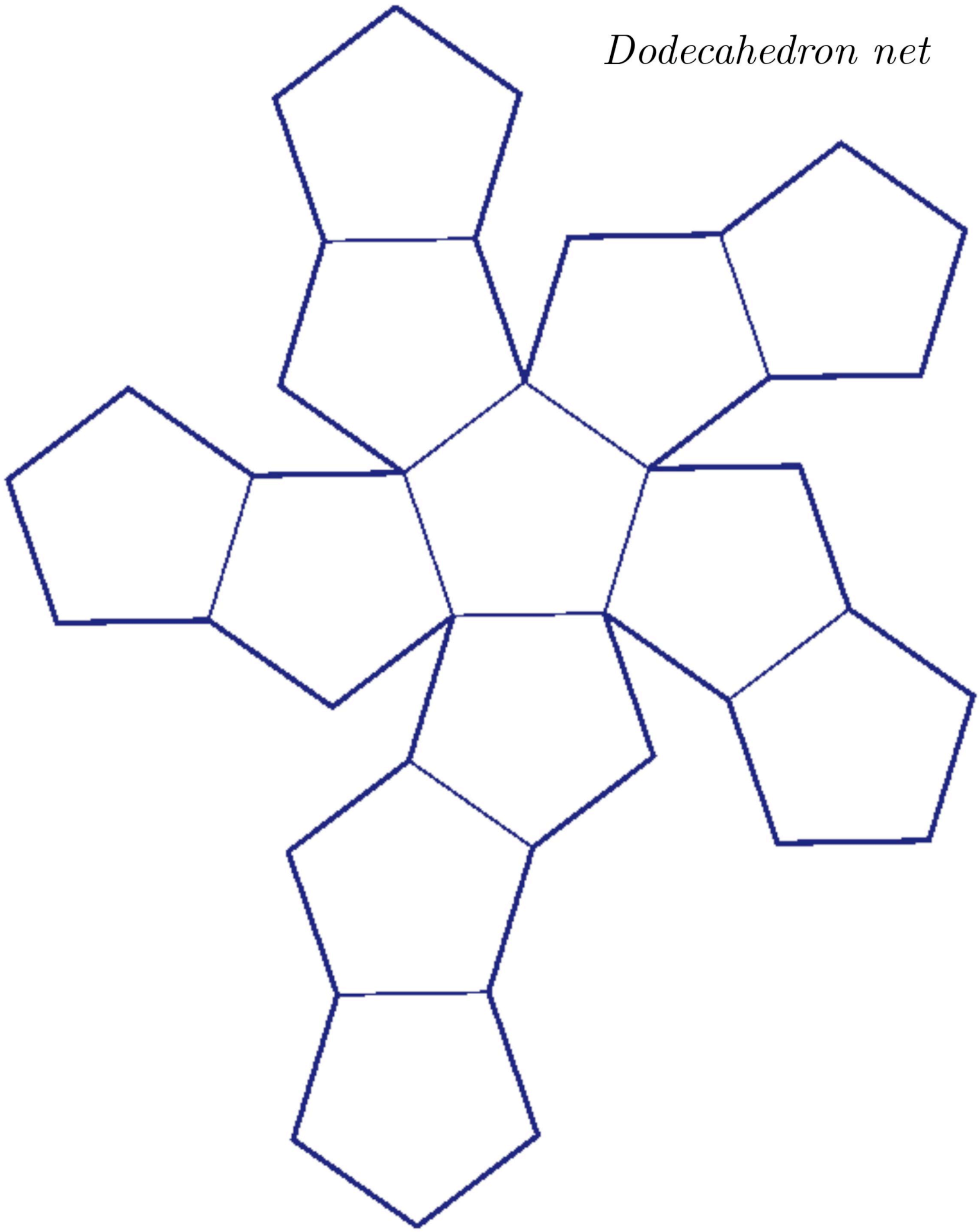


Cube net

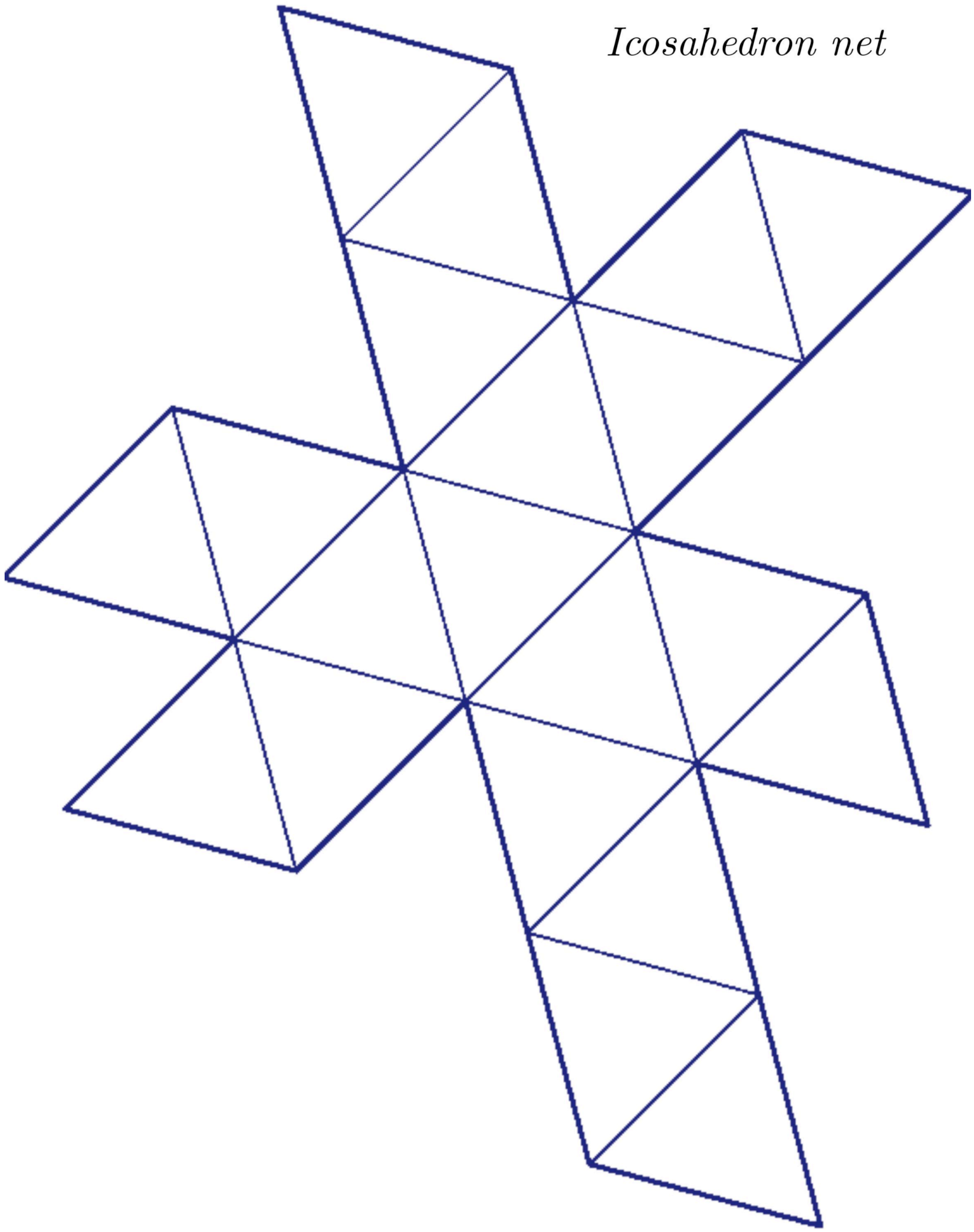
Octahedron net



Dodecahedron net



Icosahedron net



"Polyhedra" is a Greek word meaning __________ ____________.

A ________________ is a three-dimensional figure formed by regions shaped like polygons that share a common side.

A ______________of a polyhedron is a flat surface formed by a polygon.

An _________________ of a polyhedron is the line segment where two faces intersect.

A _______________ of a polyhedron is the point at which three or more edges intersect.

A polyhedron is _____________ if all faces are congruent regular polygons and all faces meet at each vertex in exactly the same way.

What is the measure of each interior angle of an equilateral triangle?

What is the least number of equilateral triangles that can come together at each vertex to form a solid?

What is the maximum number of equilateral triangles that can come together at each vertex to form a solid?

What is the measure of each interior angle of a square?
How many faces come together at each vertex of a cube?

What is the measure of each interior angle of a regular pentagon?

Can you devise a formula for finding the interior angle of a polygon and express it in terms of the sides of that polygon? Explain your thinking.

How many regular pentagons can be put together at a vertex to form a solid?

Briefly explain why do you think there cannot be more than five Platonic solids.

Count the number of faces, vertices, and edges for each of the platonic solids. Complete the table below.

	Faces	**Vertices**	**Edges**
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

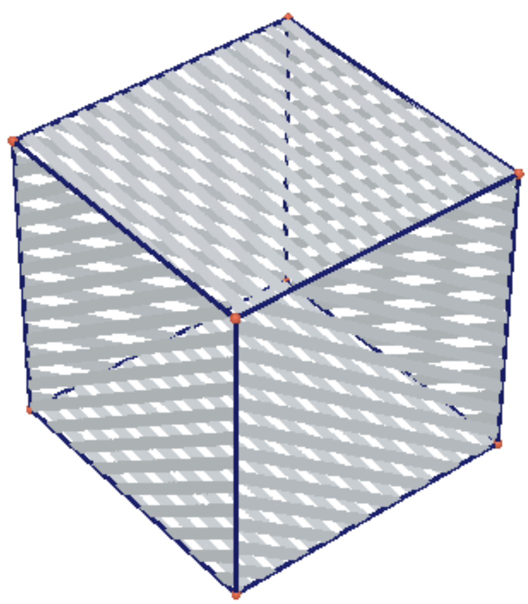
Describe the algebraic relationship that exists between the sum of the faces and vertices and the number of edges.

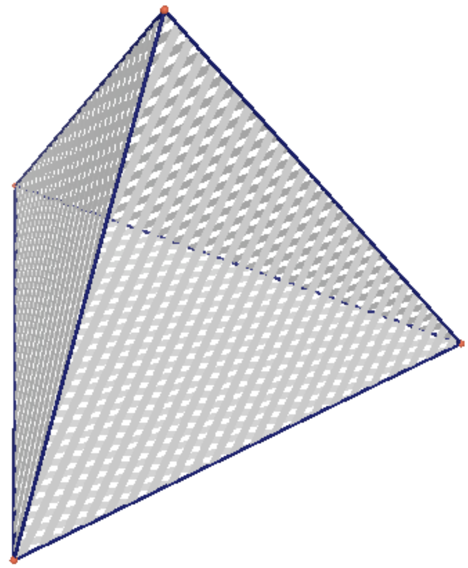
Complete a table for Archimedean polyhedra and see whether your formula works there. Write down your observations.

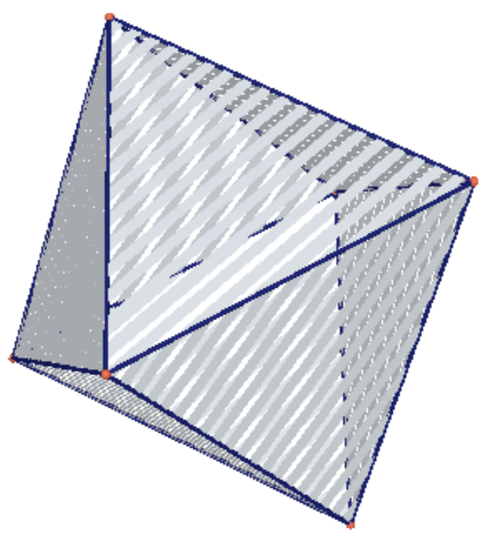
		Faces	**Vertices**	**Edges**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				

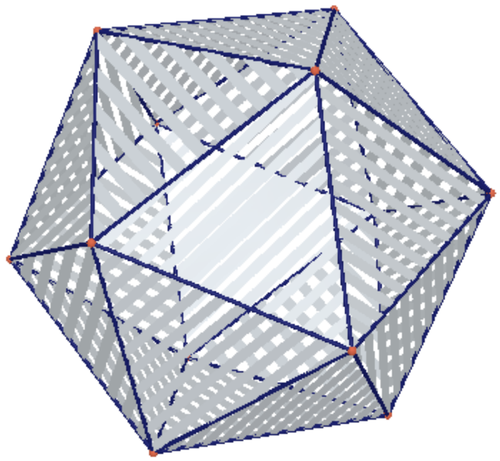
Dual polyhedra

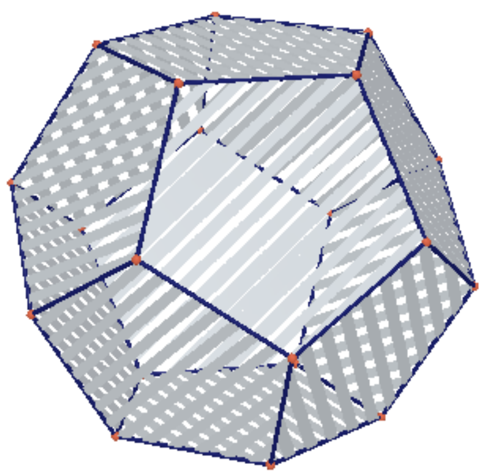
A *dual polyhedra* is made when you replace vertices with faces (and vice versa). You are here given pictures of the five regular polyhedra. Using pencil (you may use a ruler, but you may decide it is easier just to try to sketch freehand) try to draw duals of the given polyhedra. Start by making polygons around the vertices of the polyhedron. Vertices of the new, dual polyhedron will lie exactly above the midpoint of the polygons of the given polyhedron.











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