Full, original, version of the review. The finally accepted, abridged, version will appear in the *British Journal for the History of Mathematics,* 2020, vol.35 issue 3
https://www.tandfonline.com/toc/tbsh21/current

Cullen, Christopher. *Heavenly Numbers. Astronomy and Authority in Early Imperial China*. Oxford, UK: Oxford University Press, 2017.

Christopher Cullen began his scholarly career as a specialist on the subject of early Chinese astronomy and mathematics with a paper published in the *Bulletin of the School of Oriental and African Studies* in 1976: "A Chinese Eratosthenes of the flat earth: a study of a fragment of cosmology in *Huai nan tzu*." As Cullen explained, he had been inspired to study a short passage of the ca. 120 BC 淮南子 *Huai nan zi* thanks to a "tantalizing reference" in Needham (SCC, III, 224). What Needham noted there was the fact that the *Huai nan zi*, an Early Han Daoist collection of essays on a wide variety of subjects, included an account of how gnomons and their shadows could be used to determine the height of the heavens, i.e. the distance to the sun from the (presumed to be flat) earth. Needham pointed out the similarity of the method used in the *Huia nan zi* to the one Eratosthenes had employed using two gnomons at Alexandria and Syene, a known distance to the south, to calculate the circumference of the of the earth, assumed to be spherical. The challenge Needham posed—that the account in the *Huai nan zi* was "distinctly obscure, and no satisfactory interpretation has yet bene made"—prompted Cullen to undertake his own thorough study of the text in question:

> My first efforts to understand the original text led me to the conclusion that it was unique in a number of ways and had points of interest that did not fully appear in the version of Maspero, 1929, 348 ff. Professors A. C. Graham and D. C. Lau both gave very generously of their time in discussions of my first draft. Professor Graham raised the question of possible Mohist influences and brought to my notice a commentary by the Ch'ing scholar Ch'ien T'ang 錢塘, *Huai Nan tzu t'ien wen hsiin pu chu* 淮南子天文訓補注, c. 1788. I did not feel able to follow Ch'ien in all he wrote, but was relieved to find that we reached the same general conclusions about what the text was saying, although Ch'ien seems to miss much that is important. While there are still some obscurities of language in what appears to be a rather corrupt text, I offer the version given here with a fair degree of conviction that it does not substantially misrepresent the intentions of the unknown author [Cullen 1976: 106].

Subsequently, Cullen completed his Ph.D. thesis, devoted to "Cosmographical Discussions in China from Early Times up to the T'ang Dyansty," at the School of Oriental and African Studies, University of London, in 1977. In many respects, his thesis provided a roadmap of sorts for the course his research has subsequently followed over more than four decades of detailed scholarship, devoted largely to the subject of ancient Chinese astronomy, culminating in *Heavenly Numbers, Astronomy and Authority in Early Imperial China*. It is an impressive achievement, and offers readers a broad overview of not just the details of ancient Chinese astronomy, but its socio-political context as well, without which the achievements of the astronomers and the systems they devised to fathom the heavens cannot be properly understood or appreciated.

Early on Cullen showed an interest in using computers to facilitate historical research into the details of Chinese mathematical astronomy. In the course of research for his thesis, he enlisted the help of the University of London Computer Centre to analyze data from the 星經 *Xing jing* (Star Canon), an astrological work attributed to 石申 Shi Shen of the Warring States period. Needham relied on a study published in 1930 by the then-director of the Kyoto University observatory, Joe Ueta, in dating north polar distance measurements given in the *Xing jing* for various stars, concluding that armillary rings might well have been used for north polar distance measurements by astrologers like Shi Shen as early as the 4th century BCE. On contextual grounds alone Cullen had good reason to doubt that the *Xing jing* could be used to establish so early a date for the use of such an instrument, given that the *Xing jing* includes references to place names that were only current much later in the Sui (581–618 CE) and Tang (618–907 CE) dynasties (i.e. 6th century CE and later). But to establish his point on firmer grounds based on the data to be found in the *Xing jing* itself, Cullen went more than an extra mile, and submitted the recorded figures to a detailed check using computers. The text itself includes numerical data for 46 stars, and after identifying the actual stars in question, Cullen used the computer to determine their coordinates at ten-year intervals between 3000 and 1500 BCE, taking into account precession and variations in obliquity of the ecliptic. The computer analysis identified the epoch at which the *Xing jing* data best fit the actual positions of the stars in question, first for the given north polar distances and secondly for the given position in the specified 宿 *hsiu* (*xiu*) or lodges (sometimes referred to as "lunar mansions") in which they presumably appeared. Although the canon contains numerical data for 46 stars, only five had dates for north polar distances and lodge positions agreeing within 300 years. Cullen's concussion: "Results of this kind are quite inadequate as a basis on which to construct theories about the date at which measurements with armillary rings may first have been made…. Whatever the reason, it seems clear that there is little hope of dating the text by its numerical contents…. Evidence drawn from a book of this nature is not sufficient to revise conclusions on the subject of the first use of armillary rings" [Cullen 1977: 413-414]. Likewise, throughout *Heavenly Numbers*, Cullen makes frequent use of computer programs like Starry Night Pro™ and NASA's Horizons ephemeris software to analyze and evaluate ancient observational data and astronomical systems.

As Cullen explains in his preface to *Heavenly Numbers*, this book offers an introductory history of mathematical astronomy in China from the late third century BCE to the middle of the third century CE. This spans a period of nearly 450 years, from the Qin dynasty, when China was unified at the end of the Warring States period, to the end of the Han Dynasty in 220 CE. Cullen makes clear his aim at the beginning:

> I hope that this book will prove to be useful and interesting to historians of science worldwide, and to general historians of China, as well as to those with more specialist interests in the history of astronomy, or the history of science in China. I have therefore written as far as possible with the needs of such broad and disparate readerships in mind. [4]

To achieve this goal, Cullen lets the rich library of surviving documents discussing and detailing ancient Chinese astronomical systems speak for themselves. This is why access to the main sources on which he draws is facilitated by the companion volume to the one under review here—namely a volume that appeared not long before this one, in a new series edited by John Steele of Brown University. Thus in Cullen's *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems* (London: Routledge, 2017), readers will find in translation several of the main sources on which *Heavenly Numbers*, a narrative history of ancient Chinese astronomy, depends. Serious readers would do well to have both books at their disposal, for the documents in their rich detail support the many points Cullen makes, often with reference to the *Foundations of Celestial Reckoning*, in the course of *Heavenly Numbers*.

In Chapter 1, "The astronomical empire," Cullen stresses the ways in which astronomical expertise served different needs, those of the emperor and empire, as well as the subjects of both—from the most learned of the official bureaucrats to the lowliest of the peasant farmers. Here the most important astronomical responsibility was that of the emperors "bestowing the seasons on the people." The determination of a calendar was crucial for the machinery of the imperial state. Thus the book opens with a detailed account of their role in establishing the health and viability of the emperor and his reign. Cullen also explains the basic means of creating calendars and calculating the predictions of celestial phenomena based upon a given calendrical system.

Beginning with terminology, Cullen explains the significance of the single term *li* to which his study is primarily devoted. In any of its various forms, as 厤, 曆, or 歷 (all pronounced *li* and variously translated—often misleadingly, he notes—as "calendar," but more correctly as "sequence," "count," or even "calculate"), in general, "*Li* can also refer to the entire system of constants and algorithms that generates a calendar, and can thus be rendered as 'calendrical system', or (following Nathan Sivin) 'astronomical system'." [24]

The calendars that were issued based upon a particular *li* were important with respect to human conduct, not only for the emperor but for his subjects as well. For the emperor, the calendar would specify the beginning of the year, the changing of the seasons, and the timing of annual rituals; for individuals, the calendar might specify auspicious and inauspicious days or periods for important life events. For the emperor, the harmony of his reign depended upon an agreement between the terrestrial and celestial worlds. For the empire, the calendar could also be used to manage the people. An inaccurate calendar portended disaster for an emperor. Failure to "deliver the seasons" was taken to result in natural disasters and social disorder. Knowing the seasons, the correct timing of solstices and equinoxes, was crucial in an agrarian economy.

For example, if the astronomers failed to insert a needed intercalary month, the calendar might be significantly out of step with the actual seasons—as a passage Cullen cites from the 左傳 *Zuo zhuan* chronicle (providing rich historical coverage of the Spring and Autumn period (771–476 BCE)), attests. This includes the story of Confucius blaming the appearance of grasshoppers in winter on "a mistake by those in charge of *li*." [26]. A correct calendar, on the other hand, mirrored the seasonal rhythms and thereby made it possible to properly schedule sacrifices, rituals, and religious festivals at their appropriate times.

Writings about the heavens fall into two broad categories: 天文 *tian wen,* concerned primarily with the patterns, signs, omens, etc.—"astrology"—as opposed to 曆法 *li fa*, devoted to the methods and procedures of a given astronomical system—mathematical astronomy. *Tian wen* concerns the observation and interpretation of the significance of celestial events from eclipses and planetary conjunctions to comets and a wide variety of meteorological phenomena, all regarded as portents or omens, whereas *li fa* concerns procedures for calculation and predicting movements of the heavens and the timing of particular events, including planetary conjunctions, but especially solstices and equinoxes, thereby determining the annual sequence of days, months, and seasons, as well as such predictions as the occurrences of solstices, equinoxes, planetary conjunctions, and eclipses.

Cullen's focus in *Heavenly Numbers* is on the *li fa*—the writings that deal with how the calendars were determined, and less on their significance in predicting omens and disasters. The actual determination of calendars was the work of specialists, technical masters adept at the computations required for setting the calendars upon which the emperor and commoners alike relied, albeit in very different ways.

Chapter 2, "*Li* in everyday life: dates and calendars," examines ancient Chinese calendars themselves, the different forms of calendars, their structure, and how they functioned. In addition to explaining the different terms and divisions of calendars, the significance of lunar phases, reign titles, the sexagenary cycle, etc. Cullen shows how to correlate Chinese dates with Julian dates by means of an easy-to-follow example using Starry Night Pro™ software [58–59].

Cullen offers an equally helpful guide for how to "look" at a calendar by providing a specific example, a calendar on bamboo strips excavated in 1972 at Yinqueshan and dating to 134 BCE. After explaining the layout of the calendar, the significance of various markings to identify the seasons, equinoxes, ominous days, days for particular sacrifices, etc., Cullen goes on to discuss a particular kind of calendrical almanac of sorts, so-called day-books, 日書 *ri shu*. These provide records of daily events, marking good or bad days, listing events or rites performed, auspicious days for undertaking a journey, negotiating business deals, beginning the construction of a new house, planning a marriage, even good days for dogs, cats, sheep, or chickens might be recorded. As Cullen succinctly describes the significance of these records:

> The daybooks give us a rich fund of evidence of the ways in which ordinary people might make use of the calendar as an essential aid to the conduct of life, not only in looking ahead to plan the future, but also in dealing with the unexpected. To those above them, however, the calendar was essentially an instrument of power and authority, and a guarantee of the support of the cosmic order for the imperial state. An emperor saw these matters quite differently from a subject, and it is to the concerns of an emperor that we shall now turn [Cullen 2017: 72].

Indeed, Chapter 3, "The Emperor's Grand Inception, and the defeat of the Grand Clerk," surveys the ways in which the calendar was regarded as an instrument of the state, a forecast of the correct functioning of the state, a barometer of sorts reflecting the harmonious connection (or not) of the cosmic order with the emperor. Here Cullen recounts the first great reform of ancient Chinese astronomy, driven by the emperor 武帝 Wudi and his obsession with immortality. The story goes something as follows: in 105 BCE, the seventh emperor of the Han dynasty traveled to Shandong, to Mount Tai, the easternmost of the Five Great Mountains of ancient China and therefore associated with sunrise, birth, reincarnation. The emperor sought to carry out sacrifices on the exact day calculated to be not only the winter solstice that year, but one marking the completion of an astronomical cycle going back to the presumed Grand Origin (some four and a half millennia earlier). This was the date from which the traditional Chinese calendar was computed and on which a new calendar, it was determined, should begin—one that was expected to change radically the relations between the cosmic powers and those of the emperor's realm [98].

Emperor Wudi's search for immortality was tied to finding exactly the right calendar, and to do so competing theories were tested with careful observations intended to determine the superiority of one proposed calendrical system over others. The one eventually chosen abandoned the traditional "quarter remainder system" of all previous Chinese calendars prior to 104 BCE. Instead of a year of 365 ¼ days, the new calendar was based on a lunation of 29 43/81 days and a solar cycle of 365 385/1539 days. This system, known as the 太初曆 *Tai chu li* (Grand Inception System), was not inspired as were most reforms by growing inaccuracies of the calendar in use at the time, but was introduced for purely numerological reasons. The end of an astronomical period stretching back to the reign of the Yellow Emperor was fast approaching, and it was deemed auspicious to begin a new calendar on the date when the heavens were calculated to realign with the exact same positions assumed to have been those marking the beginning of the Yellow Emperor's reign, namely when the winter solstice was due to fall in the 11th Xia month of the year 105 BCE. By emulating the same rites the Yellow Emperor was said to have performed on Mount Tai at the inception of his reign, the emperor Wudi hoped that he too would thereby achieve immortality. The emphasis of the related reform of the calendar was on what was taken to be the auspicious number 81, a number associated with the harmonies of the Yellow Bell pitchpipe. Thus the number 81 was chosen to serve as the denominator of the day fraction in computing lunations, taken now to be 29 43/81 days, a number, Cullen explains, that had the practical effect of slightly increasing the length of the solar cycle.

However, the new astronomical system was not as accurate as it was not doubt intended to be. Cullen uses astronomical software (StarryNight Pro$^{TM}$) to determine that the new system did not actually coincide with the true astronomical alignments at Chang'an, then the capital of the empire. In fact, as Cullen emphasizes, throughout the Western (Former) Han, conjunctions were generally predicted about a day too late—frequently occurring on the last day of the lunar month rather than on the first day when any eclipse of the sun should have occurred. This situation was not really corrected by the Grand Inception reform. Moreover, the determination of when the new system should begin had already been determined in 113 when the conjunction was calculated using the old system. The new system was adopted, as Cullen says, because "it was the emperor's wish to emulate the Yellow Emperor in attaining immortality that was the real driving force behind the reform." [105]

The new system was not without its critics, and Cullen cites an incident in 78 BCE, quoting from a memorial written by the Grand Clerk 張壽王 Zhang Shouwang to the emperor:

> The astronomical system is the great thread guiding heaven and earth, made by the Supernal Lord. From the beginning of the Han [up to the Grand Inception reform], the dynasty made use of the Yellow Emperor's *Tau lü li* 調呂曆 "astronomical system harmonizing the standard pitchpipes." Now [under the new system] *Yin* and *Yang* are no longer in harmony, and it is time to put right the faults in the system [*Han shu* 21s, 978; Cullen 2017, 371]. [109]

To settle the matter, 鲜于妄人 Xianyu Wangren, a court calendrical expert, was instructed to carry out observations of his own with a group of several dozen from the astronomical bureau in order to compare Zhang's claims against those of various *li*, and in surveying data from 78 to 74 BCE, Xianyu's observations not only vindicated the Grand Inception system as the most accurate of all the systems tested, but demonstrated the inaccuracy of the *Tau lü li*, which was found to be seriously in error.

Chapter 4, "The Triple Concordance system and 劉歆 Liu Xin's 'Grand Unified Theory'," presents the first astronomical system "embodied in a detailed text with full instructions for its use," namely the 三統曆 *San tong li* (Triple Concordance system) due to Liu Xin. With the fall of the Western Han dynasty in 9 CE, when 王莽 Wang Mang, a Confucian determined to reestablish the harmonious times associated with the ancient classics and the legendary Yellow Emperor, the creation of a new calendar was a matter of serious dynastic importance. Wang even claimed to be descended from the Yellow Emperor, and sought to restore the golden age of the Zhou dynasty by instituting new standards and policies conforming with those of ancient Zhou times. By introducing a new calendar, he expected to solidify his position as the one to reestablish harmony between the terrestrial and cosmic realms.

In the case of Liu Xin, he used the basic system of 104 BCE as the foundation for a new calendar for which all the major constants governing the motions of the heavens were derived from numbers associated with the ancient divinatory handbook, the *Yijing*. Cullen likens Liu Xin to Kepler, both of whom believed that the cosmos was inherently mathematical. In Liu's case, he retained some basic features of the Grand Inception scheme, including its values for the lengths of the lunar and solar cycles. He also took the winter solstice of 105 CE as the starting point for calendrical calculations. In addition to predicting the positions of the sun and moon, his system sought to account for the motions of the five planets as well, something the Grand Unified Theory had not done. This meant that in order to find an inception date for the new system, he had to set a beginning date that was considerably earlier in order to find a time at which not only the Sun, Moon, and winter equinox coincided, but each of the five planets also had to be aligned. Thus, whereas the cycle of the Grand Inception system was taken to be 4,617 years long, the new system had to occur at the winter solstice over a cycle of some 143,232 years, as Cullen computes it. Liu Xin intended, as Cullen puts it, to provide "a theory of everything," all based on numbers, for as he notes:

> [Liu Xin] appears to have been determined that the basic planetary constants he used should have mathematical properties that he saw as being of cosmic significance, and rather than selecting a single set of observation data to match, he set out to match a whole range of historical records. All this was, however, part of a much wider project to use numbers to describe not just the way the celestial bodies behaved—but the way everything behaved [Cullen 2017: 133].

As Liu Xin explained his aims: "In deriving astronomical systems, producing pitchpipes and making vessels, encompassing the circle and setting right the square, weighting the heavy and balancing what is equal, levelling the [builder's] line and making fair the capacities, … there is nothing in which [numbers] are without application." [136] Basically, Liu Xin sought to show that all possible regularities in the world could be explained quantitatively in terms of numbers, a view close to views held by the early Pythagoreans.

The *San tong li* included a list of 21 concordance constants that were used to calculate solar and lunar cycles, including data needed to predict lunar eclipses. Also, appropriate constants for the planets made it possible to determine the positon of a given planet on any given date. In carrying out his own computations to check the accuracy of the *San tong li*, Cullen found that the model is "quite well followed in detail by the predictions of the Triple Concordance." [155] In fact, using Noel Swerdlow's program, "Planetary, Solar and Lunar Visibility" (PLSV), in every case the Triple Concordance prediction of the date when a planet would first be seen at dawn was not far from modern predictions. Although in general they proved to be a few days late, as Cullen points out: "the effect of such a delayed prediction would have been to ensure an increased chance that the planet would indeed have been seen in the east if an observer were to look for it on the day predicted by the Triple Concordance." [157]

A major task of any system was to determine the days on which the months of the year should begin, complicated by having to know how many years had intercalary periods. In addition to finding the day of the winter solstice, which depends on the solar cycle, it was necessary to predict lunar eclipses which could only occur if a full moon was close enough to the point where the moon crosses the ecliptic, which happens twice in every circuit of the heavens. However, no prediction based on simple cycles can take account of all the complex changes in the speed of the moon and the varying positions of its orbit.

Cullen again takes to computations to examine the first 23 eclipse predictions made by the Triple Concordance system after the Grand Inception reform of 104 BCE. In doing so, he found that in only three cases was nothing at all to be seen of the eclipse near the predicted date; in about half of the cases, the eclipse would have been observable, weather permitting. As a result, Cullen concludes that the predictive ability of the Triple Concordance system was a “moderate success.” [169]

Particularly interesting are Liu Xin’s efforts to confirm the accuracy of his Triple Concordance system, which he does in a text Cullen ascribes to him that follows immediately after the text of the *San tong li* in the *Han shu*, namely the 世經 *Shi jing* (Canon of the Ages), a document, Cullen says, that “goes systematically through a list of rulers from high antiquity onwards, and discusses astronomical and calendrical data from their reigns.” Much of the data is drawn from the 春秋 *Chun qiu* (Spring and Autumn Annals), dated events that Liu Xin uses in his attempt to determine the accuracy of the Triple Concordance System. Additional tests are made with dates from remote antiquity that were known to him from other historic documents as well, like the following from the 國語 *Guo yu* (Conversations from the States) dating to the late 4th century BCE:

> Formerly when Wu Wang attacked Yin, the Year [star] (i.e. Jupiter) as at [the station] Quail Fire, the moon was at Celestial Team, the sun was at the ford of Ximu, the conjunction was at the handle of Dipper, and the star […Mercury] was at Celestial Tortoise. [175]

Taking the date of the conquest mentioned here as 1122 BCE, Liu Xin reconstructed an entire set of corresponding astronomical conditions with “some exactitude,” says Cullen, based upon the Triple Concordance system. The fact that the actual date of the Zhou conquest is believed by most scholars to have been at least a century later, and since it is not clear exactly how Liu Xin made his calculations, at best what can be said is that he took the accuracy of the Triple Concordance system with respect to known historic events as a test of the correctness of the system. He did the same for other dateable ancient celestial events from the *Spring and Autumn Annals*, for example, the well-known date of the fifth year of the Duke Xi of Lu (655 BCE), when the preceding winter solstice was observed on the same day as there was a conjunction of the sun and moon (Cullen dates this to December 25, 656 BCE).

Nevertheless, because solar eclipses are infallible indicators of a solar-lunar conjunction, Cullen considers three datable eclipses mentioned in the *Chun qiu*: in 546, 511, and 505 BCE. All three correspond to verifiable eclipses—for which Cullen admits Liu Xin faces “major problems,” since the Triple Concordance system predicts no conjunctions that fit anywhere near the months in which these eclipses occurred [176]. Cullen’s conclusion:

> Despite the inconsistencies and loose ends in the *Canon of the Ages*, Liu Xin’s attempt at systematic checking of the astronomical events recorded in major historical sources was a major departure. It was certainly successful enough to set the pattern for many subsequent attempts, particularly in relation to the solar eclipses of the *Spring and Autumn Annals*. [178]

In the centuries to follow, the means by which the heavens were both conceived and measured underwent dramatic changes, as Cullen explains in Chapter 5, “The measures and forms of heaven.” Here Cullen describes his own efforts to recreate some of the instruments and observations that were made, intended to determine the size and shape of the heavens and earth. Cullen also offers compelling portraits of those who actually made the observations and analyzed their own data as well as the substantial data recording astronomical phenomena preserved in historical documents. The first of the scholar officials Cullen presents is 桓譚 Huan Tan, who lived at the time of Wang Mang and into the early Eastern (Later) Han dynasty and was a friend of the imperial astronomer Liu

Xin. Of particular interest here is a debate Cullen recounts between Huan Tan and the writer-philosopher 揚雄 Yang Xiong over the shape of the cosmos.

This in turn leads Cullen to consider how evolving ideas about the heavens were reflected in new instruments and different ways of measuring the positions and motions of the heavens. The basic instruments at Huan Tan's disposal were the gnomon for measuring the sun's shadows and the clepsydra for measuring time. Central to Huan Tan's survey of the heavens was the system of 28 lodges or divisions of the sky. The lodges were correlated with the seasons and associated with the expected behavior of wildlife at different times of the year, the sacrifices to be made in a given month, and even the appropriate colors for ritual robes. Each of the 28 lodges is associated with an asterism, and among the important data recorded were the lodges in which the sun was located at any given time, also with the corresponding lodges "centered" at dusk and dawn.

In June of 1979, while at Clare Hall, University of Cambridge, Cullen reenacted for himself the sorts of observations Huan Tan had made several millennia earlier. Using a 2-meter gnomon and a make-shift clepsydra, Cullen found that by observing centered stars, these provided an easy-to-follow sequence through the solar cycle whereby, if the appropriate stars for a given month were not centered at dusk or dawn, this would indicate the need for an intercalary month to allow the calendar to get back in step with the heavens. "Thus the calendar could be managed quite accurately without calculation, and with only the simplest of observations." [193]

Several recently excavated objects from Han dynasty tombs are of particular relevance to Cullen's discussion of ancient Chinese cosmology. One is an object found in the tomb of the Marquis of Ruyin (ca. 165 BCE), which consists of two lacquered wooden plates, one square, the other circular, along whose circumference appear the names of the 28 lodges as given in the *Huai nan zi* (see Cullen's Fig. 5.7, below left). The same lodges appear on the outer square earth-plate, which is further divided into four groups of seven corresponding to the four cardinal directions, north, south, east, and west. The circular plate was meant to rotate on a small pin set in the center of the square plate. Such artifacts were widely used for divination, but also reflected a Chinese model of the cosmos. As Cullen quotes from a fourth-century poet, Song Yu: "The square earth is my chariot, and the round heaven is my canopy." [203]

Another object known as the "Lodge Dial," found in the same tomb, presents the lodges spaced according to the number of degrees or *du* required for each lodge (which could range from 2 to 33 *du*) (Cullen's Fig. 5.8, below right). As Cullen surmises, this could actually be used as a rough analog computer to determine the lodge, for example, in which the sun would be centered at a given time.

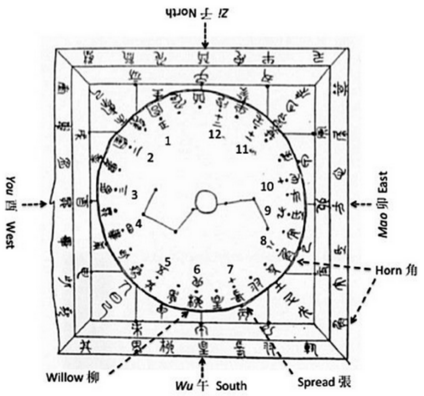



Figure 5.7 Cosmic model shi frm the tomb of the Marquis of Ruyin (165 BCE) [Cullen 2017: 203].

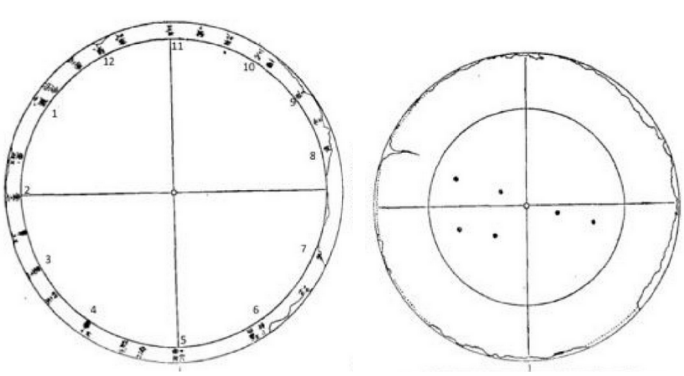


Figure 5.8 "Lodge Dial"; the smaller disc (right) was meant to rotate inside the larger disc [Cullen 2017: 206].

These models reflect the Chinese view of the size and shape of the cosmos according to the 蓋天 *gai tian* (umbrella heaven) theory recorded in the *Zhoubi suanjing* (Mathematical Classic of the Zhou Gnomon). The 蓋 *gai* of *gai tian* refers to the large umbrella-like covering of a Han period chariot, and thus the heavens were taken literally to be shaped like an umbrella. On the assumption that the earth is flat, from a suitable measurement of the length of the sun's shadow it was determined (in the *Zhoubi suanjing*) that the sun must be 80,000 *li* above the square earth.

Given the available data, Cullen surmises that one significant innovation that seems to have occurred by the Eastern Han dynasty is the use of an instrument to measure angles, something akin to a graduated armillary sphere or astrolabe. This would have enabled ancient Chinese observers to record the north polar distances of the sun in different lodges, and from such a device Cullen suggests that a correspondingly different view of the heavens emerged—not one of an umbrella but of a sphere, with the observer located at its center. This alternative to the *gai tian* theory appears in Eastern Han documents as the 渾天 *hun tian* theory (spherical heaven).

To bring his account of the *gai tian* and *hun tian* theories to a close, Cullen recounts the story of a proponent of the *gai tian* theory, the poet Yang Xiong, who was said to have carefully checked the theory against the occurrences of the four seasons, as well as the timings of sunrise and sunset, and based upon data collected from gnomon and clepsydra measurements, he drew a diagram of the *gai tian*, umbrella-shaped heavens, against which 桓譚 Huan Tan raised two objections. One concerned the problem of accounting for the equal hours of night and day at the equinoxes were the *gai tian* theory true, and the other was the impossibility of the sun ever appearing directly overhead if it were just moving horizontally as the *gai tian* umbrella rotated. In the end, Yang Xiong must have been persuaded, because Cullen quotes from Yang Xiong's 法言 *Fa yan* (Exemplary words) as follows:

> Someone asked about the *hun tian.* [Yang Xiong] said: "Luoxia Hong devised it, Xianyu Wangren calculated it, an the Palace Assistant Geng made an image of it. So subtle, so subtle! No one can contradict it! [Cullen 2017: 221]

This leads Cullen to investigate official documents revealing how experts dealt with new astronomical proposals and how they were subjected to tests of their predictive accuracy. Thus in Chapter 6 he explains the need for a new astronomical system when the Eastern Han dynasty was established in 25 CE following the downfall of Wang Mang and his Xin dynasty. Earlier, the first reform of the calendar had resulted in the Grand Inception system of 104 BCE, followed by Liu Xin's Triple Concordance System of 9 CE, conceived by the astronomer Liu Xin largely in numerological terms through which all of the regularities of nature, both celestial and terrestrial, were to be understood in terms of the mystical properties of numbers. Liu Xin's system persisted for a number of decades, but by 62 CE discrepancies caused enough concern among court astronomers that an alternative system, a quarter remainder system, was again put into use for certain computations, although no definite system origin had yet been determined, so that its use was for a time limited. But for both practical and theoretical reasons, in 85 CE a new system was created by 編訢 Bian Xin and 李梵 Li Fan; nearly a decade later, Jia Kui advocated further alterations, inaugurating a significant change in that he used the ecliptic as the basic referent for his astronomical observations and calculations.

Here the written record to be found in the 後漢書 *Hou Han Shu* is particularly detailed, and includes an account of ancient astronomy by two court astronomer experts, 蔡邕 Cai Yong and 劉洪 Liu

Hong. By the beginning of the Eastern Han dynasty, it had become clear that the Triple Concordance system was no longer accurate, and that it "lagged slightly behind Heaven, [so that] the conjunction fell in advance of the [predictions of] the system" [Cullen 2017: 225]. Not only were eclipses of the moon made too early, but the system also failed to predict accurately crescent and full moons. Eventually, mounting discrepancies between predicted dates of the winter solstice and locating the sun in the wrong lodge concerned the emperor sufficiently that a new system devised by Bian Xin and Li Fan, a quarter-day system, was promulgated in 85 CE.

The new Han Quarter Remainder system departed from the Triple Concordance system, in which all constants were specified before computations were made, by giving computation procedures for the sun and moon first, and thereafter additional constants relating to computing planetary positions were specified. Also, planetary computations were made with respect to conjunctions with the sun, rather than from first visibility at dawn. The most important change, however, was the determination of a new system origin, two in fact, one for determining basic calendrical phenomena, and another for computing forecasts for planets and lunar eclipses.

In 85 CE the court astronomer 賈逵 Jia Kui was assigned the task of evaluating the newly-revised quarter-remainder system, which he did with a group of experts who made careful observations over the next five years. Historic records also enabled Jia Kui to survey dates of seventy solar eclipses over three centuries. The most important innovation Jia Kui introduced was his appreciation of the importance of the "Yellow Road," the ecliptic, in making celestial observations (in contrast to using the Red Road, the celestial equator). Jia Kui realized that calculating the positions of the sun and moon as if they moved uniformly with respect the equator meant that the actual sun and moon would sometimes be ahead of their predicted positions and sometimes behind them. Rather than use the Red Road for measuring degrees, it would be in better accord with the motions of the sun and moon if they were measured with respect to the Yellow Road. Jia Kui tested this idea with both recent data and data reaching back a century-and-a-half earlier and came to the following conclusion:

> As has been said, the Yellow Road has its verification, and fits together with Heaven, so that the sun has no advances or laggings, and crescent and full moons are not out by a single day. This is much more accurate than using the Red Road [as a reference], and should be put in practice [Cullen 2017, 249].

Two years after Jia Kui's death, Cullen notes an edict calling for the construction of "the Grand Clerk's Yellow Road Bronze Instrument," which Cullen takes to mean an instrument with both an equatorial ring and an ecliptic ring, although officials in the astronomical bureau apparently made little use of the instrument itself.

Among Jia Kui's many observations, he was especially interested in the highly variable motion of the moon in terms of its path whose deviations he measured both north and south of the ecliptic, as well as variations in the moon's speed, which he found to vary from as little as 12 *du* to as many as 16 *du* per day. Taking these variations into account, he was better able to predict, as he put it, "the times of occurrence of conjunctions, crescents, full moons and lunar eclipses." [264] He also verified the accuracy of his approach with respect to records of 38 lunar eclipses recorded since the start of the Eastern Han dynasty.

Cullen brings this chapter to a conclusion with the mathematician and astronomer, 張衡 Zhang Heng, famed as the inventor of a device for detecting earthquakes and whose talents extended to hydraulic engineering. Cullen presents two of Zhang Heng's important astronomical works, the first being his 靈憲 *Ling Xian* (The Numinous Explained), which provides an account of the origins of the universe, beginning from an original chaos to the creation of the heavens, including the sun, earth, moon, and planets, along with an account of their shapes and sizes. This in fact is the earliest account that survives of the *hun tian* theory with its flat, motionless earth and spherical, rotating heavens.

Zhang Heng's other work discussed by Cullen, the 渾天儀 *Hun tian yi* (Spherical Heaven Instrument), is a detailed account of the celestial sphere in terms of the equator and ecliptic, although it makes no direct reference to the construction of an actual instrument. Nevertheless, Zhang Heng managed to resolve a problem raised by Jia Kui, namely that because the imperial astronomers calculated the motions of the sun and moon as if they were moving at constant speed along the equator, this meant that their actual positions with respect to the ecliptic would sometimes be behind and sometimes ahead of their predicted positions. Zhang Heng was able to explain why this happens, and provided what Cullen describes as his "ingenious solution" to the problem of changing reference systems between the Yellow Road of the ecliptic and the Red Road of the equator. Liu Hong gives a brief account of all this with respect to his astronomical system, the new 乾象曆 *Qian xiang li* (Supernal Manifestation system).

Cullen devotes Chapter 7, "The age of debates," to examining the arguments over calendars and reforms that arose during the Eastern Han dynasty, some before large audiences of bureaucratic officials. These are of considerable interest because such debates over computations of calendars were unique to China, and no records survive elsewhere in the ancient world of anything like the debates Cullen describes in considerable detail. What makes the records of these debates so valuable are the detailed accounts they offer of the individuals, instruments, texts, rules of procedure, and even the systems of "mathematical software," i.e. the procedural *li* of the various Chinese astronomical systems.

These debates were the result of a unique Han institution, the 議 *yi* (deliberations), formal gatherings called by government officials to consider important decision-making issues of the day. One such *yi* was convened in 175 CE to consider the problem of deciding upon setting a calendrical system origin, the instant of time from which a system should be set in motion, and from which time all of the cyclical periodicities of heavenly motions were to be measured. System origins were taken to begin on dates of great conjunctions, for example at the winter solstice with the conjunction of the sun and moon coinciding at midnight, taken as the beginning of the first day of the first celestial month. Other system origins might require the alignment of not just the sun and moon, but of all the planets.

An important realization of ancient Chinese astronomers was the fact that due to all of the irregularities to which celestial motions were subject, no single system could be expected to remain in harmonious synchrony with the heavens forever. Cai Yong was among those involved in the debate of 175 CE who recognized that: "the motions of the Three Luminaries, in their slowings and accelerations and their advances and retardations are not necessarily as one. When experts try to chase after them through calculation, all they can do is to seek a fit at the corresponding time." [317]

Cai Yong was interested in defending the Han Quarter Remainder system from those who argued that its date of origin was wrong, namely Feng Guang and Chen Huang, who supported the *Yin* system. While granting that the *Yin* system must have been accurate at the time it was first put into place, accumulating inaccuracies eventually required that it be replaced. The only reliable means of determining which system should prevail at the time, Cai Yong insisted, was to resort to instruments and careful observations of the heavens. To make his point, Cai Yong simply pointed out that the official Han Quarter Remainder system successfully predicted the correct New Year's Day for the year 176 CE, whereas the *Yin* system placed it two days later. To clinch his argument, Cai Yong then issued a formal challenge—that Feng Guang and Chen Huang should make their own observations to verify the positions of the "Three Luminaries." "[But] when I raised objections to Guang and Huang, all they did was to quote the Charts and Oracles, and their answers were not satisfactory." [322]

In the end, Cai Yong was vindicated, and what was most important in settling the matter was his appeal to instruments used to provide data, and direct observations of phenomena, both taken together serving as the basis for making a final judgment. As Cullen concludes this chapter: "The ability to settle disputes by reference to the record is no doubt a formative factor in the way debates on astronomical issues were structured and conducted in imperial China." [324]

*Heavenly Numbers* comes to an end with Chapter 8: "Liu Hong and the conquest of the moon." It was the astronomer 劉洪 Liu Hong, a colleague of Cai Yong, who created the last great astronomical system Cullen considers—the 乾象曆 *Qian xiang li* (Uranic Manifestation system). This was the first system to give a complete account of the main irregularities of lunar motion, and was the result of careful analysis of the massive data available about the moon from previous Han records, thereby demonstrating as Cullen says: "the full power of the style of algorithmic modelling that was characteristic of the East Asian tradition." [325] Liu Hong had participated as a technical expert during the *Yi* debates over the calendric systems, and having worked with Cai Yong collecting the documents that form the history of astronomy in the *Hou Han shu*, he was well-acquainted with the available data and realized it was necessary once again to revise the calendar because the then-current system "was falling behind the heavens." [328]

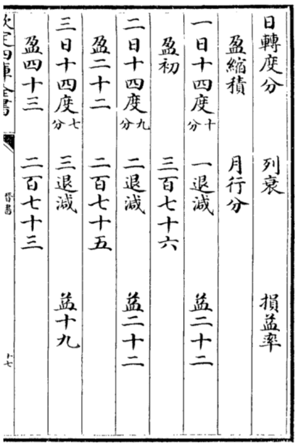
The successors to the Han dynasty during the Three Kingdoms period—Wei in the north, Shu in the West, and Wu in the east—adopted different calendrical systems. In the west where the Liu family (the imperial clan of Han) remained in power, they continued to use the Han Quarter Remainder system. But in the east, in the kingdom of Wu, the Sun clan (rulers of Wu) adopted Liu Hong's Uranic manifestation system, finding it "subtly marvelous." The astronomers of Wu used this innovative system, and made an instrument that could then be applied using it in making their observations of celestial phenomena with respect to the Yellow Road [329].

It was the successful prediction of a lunar eclipse by a commoner, 宗紺 Zong Gan, who criticized the Han Quarter Remainder system for predicting an eclipse for the following month too late, that won him an appointment as "Expectant Official," and his methods were officially adopted. When Zong Gan's grandson, Zong Cheng, having inherited his grandfather's system, said that it required revision, he predicted an eclipse that once again occurred whereas the one set by the official system did not. This launched yet another dispute that culminated four years later, in 179 CE, when Zong Cheng predicted a lunar eclipse again at variance with what the official calendar predicted. This time bad weather made it impossible to observe whether either prediction was correct or not, and debates over both again ensued. Memorials were presented to the throne arguing for one side against the other, whereupon an official inquiry was demanded and a group of four, including Liu Hong, was appointed to "comprehensively review the observation records, and deliberate equitably on disputed questions." Eventually, after further disagreements and arguments on both sides, Liu Hong's recommendation in favor of Zong Cheng's system was accepted. [333]

At this point another commoner expert, 王漢 Wang Han, arrived at court, also in 179 CE, with his own notes of observations of 96 lunar eclipses going back nearly a century, presumably collected by members of his family, upon which he based his own system origin and method for predicting eclipses. This time it was Liu Hong who was asked to look into Wang's data. Earlier Liu had criticized Zhang Xun and Zong Cheng for making adjustments as suited them, with no overall guiding principle, using instead any ad hoc means that were needed to "keep up with the heavens." As for Wang Han's system, Liu Hong recognized that the epochal origin it invoked was the same as the *Zhuan Xu* system, and in the end, he dismissed Wang Han's system as nothing new.

Meanwhile, Liu Hong had been working out the details of his own *Qian xiang li* (Uranic Manifestation system), completed Cullen suggests ca. 196 CE. Fortunately, the *Jin shu* gives a complete specification of Liu Hong's system. Li Chunfeng notes that Liu Hong correctly understood the alternating acceleration and slowing of the moon's motion, based upon a thorough examination of the *yin* and *yang* periods of the moon's path as it crossed the ecliptic, the "Yellow Road," for which Li Chunfeng praised him for understanding therefore how the moon would appear to advance and retard with respect to the "Red Road," the equator. [343] Whereas others had previously observed the moon's non-uniform motion along the ecliptic—like Jia Kui ca 100 CE, Li Chunfeng credits Lu Hong with two innovations. First there was its variation in speed with respect to the fixed stars, and then its changing distance from the ecliptic, in modern terms, its latitude. Liu Hong's methods enabled him to make accurate predictions of the times of conjunction or opposition of the sun and moon, as well as determining where the sun and moon would be at conjunction with respect to the nodes where the moon crossed the ecliptic. When conjunctions occur sufficiently close to a node, there is the possibility of an eclipse of the sun. As Cullen concludes: "Thus for the first time, specialists in *li* were able to give advance warning that a solar eclipse might take place on a given date." [344]

In the course of his study of the moon, Liu Hong charted the difference between its predicted and actual positions in a 遲疾歷 *chi ji li* ("speed sequence" table) [Cullen 2017: 345].



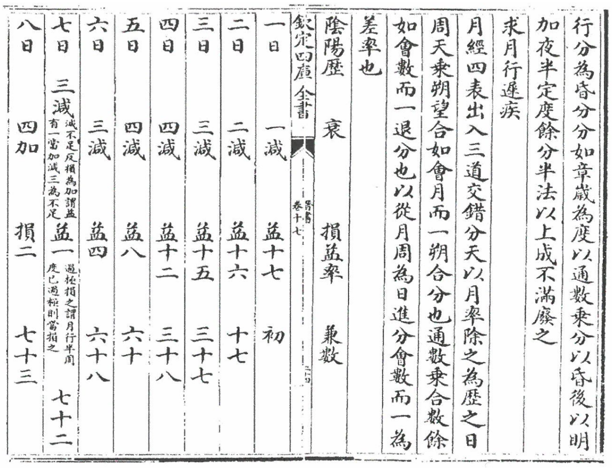
| 日轉度分 | 列衰 | 損益率 | 盈縮積 | 月行分 |
|---|---|---|---|---|
| 一日十四度十分 | 一退減 | 益二十二 | 盈初 | 三百七十六 |
| 二日十四度九分 | 二退減 | 益二十二 | 盈二十二 | 二百七十五 |
| 三日十四度七分 | 三退減 | 益十九 | 盈四十三 | 二百七十三 |

Figure 8.1. Beginning of Liu Hong's speed sequence table in *Jin shu* 17, 17a, *Si ku quan shu* edition, c. 1782 [Cullen 2017: 345].

Using NASA "Horizons" ephemeris data, Cullen tests Liu Hong's results by locating a maximum lunar speed for November 17, 206 CE, and finds that The Uranic Manifestation procedure predicts this for three days later; another six months earlier, for April 2, 206 CE, it was only off by a day, results that Cullen describes as "not obviously fatal." [350]

Before Liu Hong, Jia Kui had understood that he could use recorded observations of solar eclipses to test the validity of any given astronomical system, and he used some 70 recorded solar eclipses to evaluate different systems. He also knew that not all conjunctions resulted in a solar eclipse, but had no way of distinguishing conjunctions that would definitely result in an eclipse from those that would not. Liu Hong's methods changed this by approaching the matter of conjunctions in a completely new way.

While the sun moves eastwards at a speed of 1 du per day relative to the celestial sphere, the moon is moving nearly 13 times faster. As the moon crosses a node to catch up with the more slowly moving sun, the distance it must travel to catch up with the sun and the distance the moon is from the sun at the time of conjunction, its latitude, must be small enough so that the moon at the moment of conjunction will be sufficiently close to partially if not completely obscure the sun. [354] Liu Hong assembled his data for the moon's motion in a *yin-yang li* (*yin-yang* sequence), displayed as a table giving the day to day changes in latitude of the moon, i.e. its distance from the ecliptic, in units of 1/12 *du*.



行分為昏分分如章歲為度以通數乘分以昏後以明
加夜半定度餘分半法以上成不滿廢之
求月行遲疾
月經四表出入三道交錯分天以月率除之為歷之日
周天乘朔望合如會月而一朔合分也通數乘合數餘
如會數而一退分也以從月周為日進分會數而一為
差率也

欽定四庫全書　晉書 卷十七

| 陰陽歷 | 衰 | 損益率 | 兼數 |
| --- | --- | --- | --- |
| 一日 | 一減 | 益十七 | 初 |
| 二日 | 二減 | 益十六 | 十七 |
| 三日 | 三減 | 益十五 | 三十七 |
| 四日 | 四減 | 益十二 | 三十八 |
| 五日 | 四減 | 益八 | 六十 |
| 六日 | 三減 | 益四 | 六十八 |
| 七日 | 三減 減不足反損為加謂益有一當加減三為不足 | 益一 過極損之謂月行半周度已過極則當損之 | 七十二 |
| 八日 | 四加 | 損二 | 七十三 |

[Cullen 2017: 355]

Although it is often said that the first Chinese predictions of solar eclipses are to be found in Yang Wei's 景初曆 *Jing chu li* (Brilliant Inception [reign period] system), adopted by the kingdom of Wei and in use from 237 CE (and then elsewhere until 451 CE), Cullen argues that it was Liu Hong who first made predictions of solar eclipses. He bases this conclusion on the fact that Liu Hong had a means of computing not only the times of possible conjunctions but the corresponding latitudes of the moon at such moments as well.

The major advantage of the Uranic Manifestation system over the *Jing chu* system was its ability to determine actual values of lunar latitude from the *yin-yang* sequence. [372] The important question Cullen then raises is whether or not Liu Hong ever used his system to predict solar eclipses? He concludes that in all likelihood Liu Hong must have, given that he tested his theory of lunar eclipse predictions against nearly 30 years of previously recorded lunar eclipses. Yuan Shansong and Xu Yue both add that Liu Hong spent years testing his Uranic Manifestation system "against the sun and moon." As Cullen says:

> On the basis of his success in predicting all recorded eclipses, it would have been reasonable for Liu Hong to conclude that his system was able to indicate when a solar eclipse was likely to be seen *somewhere*—even if the official sky-watchers in the capital were unable to see it. … [H]e was entitled to conclude that he had "found [his system] to correspond to the phenomena" as Yuan Shansong said he had [Cullen 2017: 379].

Cullen concludes that Liu Hong had assembled all of the basic theoretical equipment needed for predicting solar eclipses, including:

…a method of calculating what is in effect the distance of the moon from the node in longitude at the moment of true conjunction, and of finding its latitude at that instant, as well as eclipse limits which add a convenient means of estimating the likelihood of eclipses taking place. Since he can calculate lunar latitudes, Liu Hong's methods are actually more sophisticated than those in the *Jin chu li*, which lacks this feature but up to now generally has been identified as the first system to attempt to predict solar eclipses [Cullen 2017: 373].

Assuming Liu Hong successfully tested his system against eclipse records, would this have encouraged others to make similar predictions? Cullen points out that a few years after Liu Hong had finalized his system in 206 CE, Yang Wei claimed to be able to predict solar eclipses. In fact, the Grand Clerk of Wei was using the *Jing chu* system—based as Xu Yue said, on Liu Hong's system—to predict solar eclipses, although not always successfully.

In his "Epilogue," Chapter 9, Cullen brings all of the threads of the story he has been weaving together in a final overall summary. Astronomical systems, he reminds us, were synonymous with establishing and demonstrating imperial legitimacy. Every emperor had a duty to "grant the seasons" to his subjects, and following the span of time for which he has been concerned in *Heavenly Numbers*, primarily the Western and Eastern Han dynasties, as successive empires came and went, from the Sui (581–618), Tang (618–907), Song (960—1279), Yuan (1271–1368), Ming (1368–1644), and finally the Qing (1644–1911) dynasties, so too did a procession of astronomical systems come and go, each created by successive generations of court specialists who studied, modified, and created new astronomical systems. [393]

This all required a host of specialists at various levels of the bureaucracy. In addition to the Grand Clerks, retinues of specialist observers, record-keepers, calculators, and omen-interpreters all worked together. Others in government service with no official responsibilities related to the astronomical bureau might also from time to time contribute their insights and expertise, at times even questioning the accuracy or legitimacy of a given system in use at the time. There were also private persons who had obtained levels of knowledge and mathematical skill that from time to time came into play. Open and critical discussion of technical matters often occurred, where the ultimate test of any particular astronomical system was its ability to match observations—both historic and contemporary. All of this was imbedded in a culture of record keeping and archiving of documentary resources that gave specialists a substantial amount of data upon which to base, analyze, test, and ultimately decide upon the suitability of the procedures, parameters, epochal dates, and other details that might affect the accuracy of any particular system.

This same rich body of material also enables modern scholars to study that record and weigh the success of these systems preserved in the historical record. Despite sweeping cultural changes—the spread of Buddhism, for example, throughout China in the period of division following the fall of the Han dynasty, astronomical systems and institutions persisted. Cullen mentions in particular Gautama Siddhārta, born in Xi'an but of Indian descent, whose transliterated Chinese name is 瞿曇悉達 Qutan Zida. He was responsible for transmitting various Indian astronomical works from Sanskrit into Chinese, including the 九執曆 *Jiuzhi li* (Navagrāha Calendar) based on the *Pañcasiddhāntikā*. For a time in the 8th century, Qutan Zida headed the imperial astronomical bureau, where he oversaw a team of scholars responsible for compiling the encyclopedic treatise, the 開元占經 *Kaiyuan zhanjing* (Treatise on Astrology of the Kaiyuan Era). Cullen notes that thanks to the historical information contained in the *Kaiyuan zhanjing*, our knowledge of pre-Tang astronomy is considerably richer. As Jean-Claude Martzloff points out, this was not a theoretical treatise, but a procedural text that listed the steps to be followed for determining such astronomical parameters as the longitudes of the sun and moon, the apparent diameter of the moon, and the duration of eclipses. This work also included a 月間量命 *yue jianliang ming* (sine of the lunar intervals), basically a table of sines:

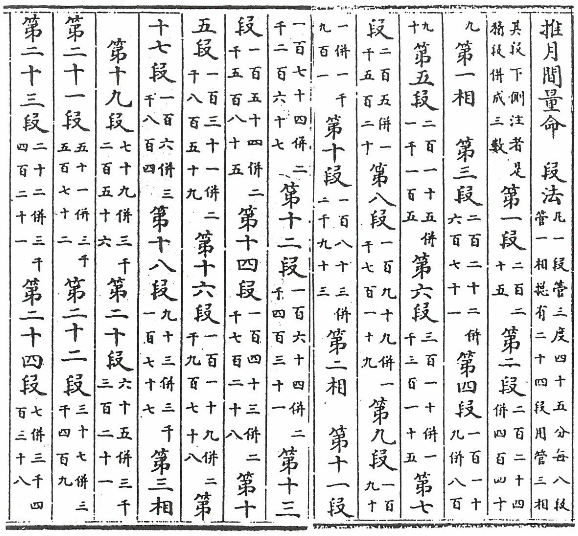


Table of sines from the *Kaiyuan zhanjing*, from the part devoted to the *Jiuzhi li*, a calendric system of Indian origin [Martzloff 2006: 101].

About this same time another important contribution to astronomy and geography in China was made by the Buddhist monk 一行 Yixing, who undertook a famous meridian survey of China. The purpose of this survey was to collect better data for improving calendars, calculating possible occurrences of solar eclipses, and determining the length of the meridian arc. Yixing was also responsible for the 大衍曆 *Da yan li* (Great Expansion system), considered one of the best calendrical systems of the Tang dynasty.

Islamic influences also made their way to China, for example during the Yuan dynasty, when 郭守敬 Guo Shoujing produced his 授时曆 *Shou shi li* (Season-granting system). Guo was a scholar official adept at creating the most accurate instruments for measuring distances and time yet devised, and these enabled him, for example, to make the best determination to date of the length of the solar year: 365.2425 days. In 1280 CE Kublai Khan adopted a new calendar devised by Guo Shoujing, which was based upon data collected from twenty-seven observatories established throughout China. The *Shou shi li* prevailed for the next 363 years, making it the most long-lived of any Chinese calendar. It was at this same time that Jamal al-Din, a Persian astronomer, was also working in the Islamic astronomical bureau in Beijing. Known in Chinese as 扎馬魯丁 Zhama Luding, he was enlisted by Kublai Khan to establish an Islamic branch of the astronomical bureau in Beijing, and expected to check the calendrical computations and calendars promulgated by the Chinese astronomers of the court.

Such foreign influences not only enriched Chinese astronomy, but when the Mongols were overthrown in 1368 CE, the Ming dynasty created a department within the astronomical bureau, the "Muslim section" (ca. 1250), to ensure that Islamic astronomy would still be available as an adjunct to the work of the official Ming astronomers at the court. Chinese translations of Islamic works into Chinese were likewise encouraged.

The Muslim section was still in operation when the Jesuits first arrived in Beijing. The ability of the Jesuits to predict solar eclipses proved to be a major factor in their success in competing with all of the other astronomers in the astronomical bureau, whether Chinese, Indian, or Islamic. The Jesuits, beginning with Matteo Ricci (利瑪竇 Li Madou), adopted Chinese ways, prepared annual calendars, and even included all of the hemerological details relating to determination of “lucky days” and, “with occasional misgivings,” as Cullen admits, also provided “interpretations of all celestial omens exactly as their predecessors had done.” (397) In the process, as Matteo Ricci’s Chinese colleague and convert to Christianity, 徐光啓 Xu Guangqi, put it, he sought to: “Melt down the substance of their [western] materials, and [cast it] into the mould of the [Chinese] *Da tong* [system]” [Cullen 2017: 397].

After 1836 CE, when the last Catholic missionaries had gone and the Qing dynasty itself had fallen, giving way to the Republic of China in 1912 CE, there was no longer an emperor to “grant the seasons,” but that did not mean an end to the Chinese calendar or to the annual need for calendars to be issued following the traditional lunar calendar. Indeed, the lunar calendar continues to be created and distributed annually throughout East Asia and indeed, wherever Chinese may have settled throughout the world. As Cullen puts it succinctly, but well: “people’s persistent demand for the ‘seasons’ in their traditional form when there is no longer any authority to grant them lends some weight to that view”—the view namely that the traditional top-down view of “season-granting” might not be the whole of the story.

Indeed, as Cullen’s book admirably demonstrates, the calendar may have served the emperor as well as the empire, but it was woven into the common fabric of daily Chinese life, and was as essential to the uneducated peasant farmer deciding when it might be best to plant or travel, as it was to the scholar bureaucrats who sought to ensure the success of the empire and the harmonious workings of state and local bureaucracies throughout the empire that was China.

In writing this book, Cullen has adopted the effective strategy of relegating necessary but avoidable technical details to separate boxed sections that those only interested in the gist of what is at issue but not the details can skip. For those who are interested in the technical richness of Chinese mathematical astronomy, however, those technical details will be welcome. This book, combined with Cullen’s companion volume, *The Foundations of Celestial Reckoning*, will not disappoint anyone with a taste for the numbers that ultimately are the point, of course, of *Heavenly Numbers*.

Cullen brings his “Introduction” to *Heavenly Numbers* to a close with a quotation from a course of lectures Otto Neugebauer once gave at Cornell University that later provided the framework for his well-known book, *The Exact Sciences in Antiquity.* There Neugebauer likened his attempt to reconstruct a picture of ancient Egyptian and Babylonian mathematics and astronomy, and the results of their transmission to the Hellenistic world, to the attempt to capture the unicorn as recorded in a famous series of tapestries at the Cloisters Museum, a branch of the Metropolitan Museum of Art in New York City. “The Hunt of the Unicorn,” a series of seven tapestries, culminates with the capture of the unicorn, “the miraculous animal captured, gracefully resigned to his fate, standing in an enclosure surrounded by a neat little fence” [Neugebauer 1969: 177; quoted from Cullen 2017: 14].

Neugebauer meant to convey that he was perfectly aware that with his version of the exact sciences in antiquity, he may only have constructed a “picture pleasing to the constructive mind” in trying to restore the past. Cullen points out, as Neugebauer did not, “that the unicorn in the picture he cites appears to be bleeding from wounds inflicted by the hunters who captured it”—and hopes that his effort will not “have done too much damage to the past that I have tried to capture in this book” [Cullen 2017: 15]. On the contrary, his unicorn has been artfully captured in this case without damage and with considerable enhancement, for here one may observe what might otherwise be a truly endangered species, namely, an appreciation of ancient Chinese mathematical astronomy. What Cullen has provided is a very readable survey of the historical development of calendrical systems in ancient China, along with the many figures, major and minor, who contributed to that development, with a lively account of the motives and results of their accomplishments. Christopher Cullen has brought the decades of his study of this difficult and arcane subject to bear on the telling of a story that is engagingly told, and intelligible even for those without a deep interest in or appreciation for the mathematics that underlies the history of ancient Chinese astronomy. But the mathematics, like the unicorn, is nevertheless on display, in neat little boxes, for all who care to follow even the minutest details.

Cullen, Christopher. *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems.* London: Routledge, 2017.

Martzloff, Jean-Claude. *A History of Chinese Mathematics*. Stephen S. Wilson, trans. 2nd ed. Berlin: Springer, 2006.

Neugebauer, Otto. *The Exact Sciences in Antiquity*. 2nd ed. Providence, RI: Brown University Press, 1957: repr. New York, NY: Dover Publications, 1969.

Joseph W. Dauben
Department of History
Herbert H. Lehman College, and
Ph.D. Program in History
The Graduate Center
City University of New York
365 Fifth Avenue
New York, NY 10016 USA
jdauben@att.net