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Robin Wilson and Amirouche Moktefi (eds.), *The Mathematical World of Charles L. Dodgson (Lewis Carroll)*, Oxford University Press: Oxford 2019, ISBN 978-0-19-881700-0.

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Lewis Carroll is famous for his literary work, in particular his Victorian fantasy tales *Alice's Adventures in Wonderland* (1865) and *Through the Looking-Glass, and What Alice Fund There* (1871). Far less known is that "Lewis Carroll" was the pen name of Charles Lutwidge Dodgson (1832–1898), a brilliant mathematician at Christ Church, Oxford University.

The volume under review is devoted to the "mathematical world" of Dodgson. The material is organized in eight chapters devoted to several fields of mathematics. The volume closes with an extensive commented "Mathematical Bibliography" of Dodgson's relevant writings.

In Ch. 1 Robin Williams and Amirouche Moktefi tell the story of "A Mathematical Life" (pp. 1–29), Dodgson's childhood in Reverend Charles Dodgson's Christian household, his education at Rugby School and his studies of Latin, Greek and Mathematics at Christ Church, Oxford University. After having received his Bachelor of Arts degree in 1854, he was awarded a Mathematical Lectureship from which he resigned only in 1881. In 1882 he became Curator of the Christ Church Common Room.

Dodgson wrote his mathematical pamphlets for helping his students. His mathematical books are directed to a wider public, among them *Notes on the First Part of Algebra* (1861), *Euclid and his Modern Rivals* (1879) and the most important of his mathematics books *An Elementary Treatise on Determinants* (1867). Dodgson's long lasting interest culminated in his *Symbolic Logic*, the first part of which appeared in 1896. He was not able to complete the second part before his death.

According to the authors' evaluation, Dodgson saw "himself primarily as a mathematical teacher who investigated some specific topics […]. Within these areas in which he believed to have some expertise, he did not hesitate to challenge other mathematicians whom he considered as much more distinguished than he was" (p. 25).

In the chapters on Dodgson's specific contributions, they are imbedded into their historical contexts using elementary examples, thus easily accessible for non-specialists.

Robin Wilson's chapter on "Geometry" (pp. 31–55), e.g., starts with a sketch of Euclid's *Elements*. Concerning the motives for Dodgson's writing on geometry the author assumes: "Although Dodgson greatly admired Euclid's *Elements*, he recognized that it contained gaps, inaccuracies, and inconsistencies. To help his pupils overcome these, he produced a number of books and short mathematical pamphlets that clarified the text, suggested alternative approaches, and included exercises for his pupils to try" (p. 39). The author relates Dodgson's work to a contemporary debate on the role and significance of Euclid's *Elements* for mathematics education. Dodgson presented his position as an outspoken advocate of Euclid in his most remarkable *Euclid and his Modern Rivals* (1879).

Adam Rice treats "Algebra" in Ch. 3, focussing on Dodgson's theory of determinants and simultaneous linear equations, which contributed to a then new direction in mathematics. These mathematical objects do not follow the standard algebraic laws like commutativity for multiplication in the case of matrices. The subject was not relevant for the Oxford examinations. The author assumes that Dodgson, in choosing the topic, might have anticipated further developments of the University's curriculum. The most original feature of his theory is the method of condensation used

for evaluating determinants. His theory was unique in respect to the logical exactness of its argument, but in its rigidity it was difficult to read, a difficulty "further compounded by Dodgson's insistence on using his own self-devised notation and terminology" (p. 79). He did, e.g., not use the word "matrix" though widely distributed through A. Cayley and J.J. Sylvester, but "block".

Ch. 4 by Amirouche Moktefi is devoted to "Logic" (pp. 87–119). Dodgson's most important books are *The Game of Logic* (1886) and his *Symbolic Logic* (Pt I, *Elementary*, 1896), both notably published under his pen name Lewis Carroll. Both books are the results of his long lasting efforts to write a "Logic for Ladies". He understood logic as a tool for the mental recreation for the youth: "If, dear Reader, you will faithfully observe these Rules, and so give my book a really fair trial, I promise you, most confidently, that you will find Symbolic Logic to be one of the most, if not the most, fascinating of mental recreations!" (p. 22, cf. Symbolic Logic, xvi–xvii). Dodgson's contributions to the emerging symbolic logic in Great Britain can be seen in his diagrammatic method which Moktefi reconstructs using elementary syllogistic procedures ("The Barbara Problem"). Dodgson was also a pioneer of the method of trees.

Ch. V on "Voting" by Iain McLean deals with Dodgson's contributions to the various methods of conducting elections, and to choice procedures for, e.g., for tennis tournaments.

Ch. VI by Edward Wakeling "Recreational Mathematics" (pp. 141–175) treats Dodgson's lifelong interest for mathematical puzzles and games. He not only intended entertaining children, but also "to test and amuse his friends and colleagues" (p. 141). Central are the "Pillow Problems": "Nearly all of the following seventy-two Problems are veritable 'Pillow-Problems', having been solved in the head while lying awake at night" (p. 24, cf. *Curiosa Mathematica*, Part II: *Pillow Problems*, 1893, Introduction).

The final Ch. 7 "Mathematical Legacy" by Francine F. Abeles (pp. 177–215) examines in sections on geometry, trigonometry, algebra, logic, voting, probability, and cryptology "the state of the subject during Dodgson's time, the important mathematical ideas and methods that he created or developed, the mathematicians that he worked with or influenced, his relevant publications and

those by other mathematicians who may have influenced him, the mathematical ideas and methods that his work foreshadowed in the 20th century, and some modern mathematicians who have been inspired by his work" (p. 177).

In sum: this is a great volume, highlighting an aspect of Charles L. Dodgson's life not familiar to the broader public. It is well accessible for the non-specialists with its contextualization of the topics treated and its elementary examples, but with its numerous puzzles and riddles sometimes challenging even for experts. Numerous illustrations, above all portraits and reproduction from manuscripts, invite the reader to dig deeper.