ROGERS REVIEW

The History of Mathematics: A Source-Based Approach Volume 1
June Barrow-Green, Jeremy Gray, and Robin Wilson (Eds.)
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Introduction

The Open University History of Mathematics courses were an outcome of the British Society for the History of Mathematics (BSHM) and the creative endeavour of a number of enterprising members of this organisation who contacted international figures in the field to write material for modules to support various themes of the programme.
The courses, entitled Topics in the History of Mathematics ran for 21 years and when they closed, the idea was to turn the whole course into book form consisting of two texts of about 500-600 pages each. This volume is the first book that covers the material available from earliest times until the last chapter entitled "European Mathematics in the Early 17th Century". The original volume *The History of Mathematics -A Reader* edited by John Fauvel and Jeremy Gray was published in 1987 by the Open University and was available to the general public, but the collection of material supporting the original course consisted not only of the 'Reader' but also a number of booklets covering special topics written by experts in particular areas of the history of mathematics. Because the course, and the books, were based on primary and secondary sources, this necessitated including a number of examples of source material, some of it adapted from the original Reader. This new publication edited by June Barrow-Green, Jeremy Gray and Robin Wilson contains a considerable amount of the material just described in some 450 pages. Also included are a number of exercises with questions on the chapters together with advice on writing an essay, which we can find here. This book is published by the Mathematical Association of America (MAA) in their 'Text Book' series.

This Volume begins with a short selective history of Egypt and Babylon through the developments of mathematics in all its forms, to the mid 17th century. This is a considerable amount of material to pack into one volume, and in terms of the time-span of the history of mathematics, it only covers about half of the material in the original Reader.
There are 13 chapters, each of varying length where the chapter headings are indicators of content, and items are arranged in approximate chronological order. However, it is impossible to avoid sections overlapping in time or content while they are dealing with

particular themes, civilisations or the influence of individuals and their ideas. The authors have had to make many choices and have done their best to cater for omissions with appropriate footnotes and collections of 'Further Reading' at the end of each chapter, with useful 'boxes' summarising information on particular topics. However, as the reader soon discovers, it would be useful to have a copy of the original Reader to hand, because many notes refer to passages found in the Reader, labelled (F&G) in this text. Many of the new references are required to bring us up to date from the material that was published some 30 years ago.

In the introduction, the reader is invited to consider a number of questions: we are warned to be careful about the use of the terms 'history' and 'mathematics'. Perennial problems, such as what is mathematics and where and how do we recognise it happening? Much of the mathematics we learn in school seemed to appear in a vacuum, so we ask: what is the problem about, who is solving problems or communicating the results, and who are they talking to and why? Ever present are questions of historiography; how has history been communicated? Can we identify different theories about the past that colour our views?

> 'History' can mean 'the past' in a simple temporal sense, or it can mean 'organised knowledge of the past', *something acquired and developed by historians*. In this book we use both meanings simultaneously: we study the mathematics of the past, and we shall also come to see how our knowledge of the past is gained - the two are inseparable. (p.6) (italics mine)

There are some particular themes that may be identified: numbers, number systems and number theories including the invention of numbers (like the zero, or 'imaginary' numbers) and the nature of 'square roots'. The nature of proof and the proving process; why is it important, in what context, and how do we agree that a proof is valid? What is the difference (if any) between discovery and invention?

Early mathematics: **Chapter 2**

The study of Early Mathematics begins with some cautions and raises issues about assumptions we might make when describing what we see. We can describe situations where investigators might see the germs of an idea when using history in a purely temporal account of events, or in the situation where we can recognise 'organised knowledge of the past'. When we see something like basket-weaving as a cultural activity, but it could also be 'mathematical', we have to make a choice and justify our decision. The point here lies in the discussion about what *we see*, and what *we interpret* when faced with a cultural artefact. Much of our knowledge of the past is mediated by artefacts, and this gets more problematic the further we look back into the past.

Egyptian and Mesopotamian Mathematics have been brought together in some thirty pages in this Chapter and both of these civilisations go back some 3,000 years: much of their history has been discovered relatively recently. There were some artefacts that had been around for some time but recent archaeology has provided much more evidence of their cultural history. The authors suggest the following exercise: (p. 29)

> *Using any means, such as any symbolism or notation that occurs to you, find your way into the problem; then check rigorously to see how much of your new understanding is a projection backwards from your own time and techniques.*

This is intended to make the reader think about what it could be like when we do not have any mathematical tools, no algebra, some geometry, and just elementary arithmetic.
The accounts here are much more sensitive to the life of the people and provide a good starting point for further investigation. Hoyrup's literal account of working with the squares on the tablet BM 13901 is challenging, but hopefully rewarding (pp. 29 – 30).

Greek Mathematics. Chapters 3, 4, 5, and 6.
These chapters take up about a third of the Book and cover the extent of the mathematics created, used and argued about from about 500 BCE (Before the Common Era) to about 800 CE (Common Era). That is about1300 years. Taken step by step, this leads the reader into the discussion of problems that became the subject of debate for many years. While knowledge of much of this material may be fairly familiar to the reader, it does not seem to me to present itself as a continuous read, and dipping into different sections could be rewarding and perhaps comforting. The certainty of knowledge, the idea of proof, the existence of objects, developing number systems, how do curves touch? What happens when two lines intersect? The lives and ideas of individual mathematicians; Euclid, Archimedes, Aristotle, Ptolemy, Diophantus, are interesting and bring personal voices to the scene.
Most important here are descriptions of the range of Astronomical observations and the publication by Claudius Ptolemy of The Great Treatise (al-Megiste or Almagest) the ancient world system which will be challenged near the end of this book.

Mathematics in India & China Chapter 7
The difficulties in investigating these civilisations has been considerable. A few artefacts and writings have been well-known, but poorly interpreted, and it is only within the last thirty years or so have serious studies been published. We should also realise that the terms we use to describe 'India' and 'China' cover vast areas of land and influence often undefined. These civilisations were often viewed as 'exotic' and little understood. Both of these sections are easily accessible to the general reader. However, there are plenty of interesting problems to 'expand the mind.'
Historians of mathematics in **India** are in a race against time. The records of ancient mathematics are disappearing since they have been written on Palm leaves that rot away in the humid climate. The history of Indian mathematics is as old as Egypt and Babylon, but those texts have been more durable. Indian mathematicians had a Base ten number system very early and this section gives us some of their knowledge. The research work of Kim Plofker is recent, authoritative, and extensive, with many good examples to explore.

The *Book of Numbers and Computations* from **China** provides the instructions for calculations, stating that this mathematics was used entirely for tax collectors. The early script numerals from about the 11th century BCE are described but the account of calculation in Chinese mathematics begins with the Counting Board and Rod numerals, showing how the base 10 system works in the visual array with the rules of elementary arithmetic. In contrast, *The Nine Chapters* is a collection of mathematical problems and procedures that cover many of the other situations we see as mathematical problems. There are many familiar contexts here for the reader; Pythagoras, Areas and Volumes, and the calculation of π.

Mathematics in Islam Chapter 8
After its foundation, the Islamic movement grew so that by the 8th century CE its capital moved to Baghdad. From this time scholars in the 'house of wisdom' travelled over the known world to collect mathematics texts which they translated into Arabic. We owe these people an intellectual debt for preserving much mathematics that was lost elsewhere. This is

where we can read of Al-Kwarizmi and his colleagues who developed and extended so much of the knowledge they had collected. The beginnings of algebra and some problems in geometry are discussed here. The story of how the mathematical texts were preserved and the ideas passed on to Europe is of particular cultural significance and deserves to be better known.

Mathematical Awakening of Europe Chapter 9
This period covers about 700 years from the end of the Greco-Roman world to the end of the Crusades about 1492. In spite of the chaos in Europe, European and Arab scholars were in contact and translations were made of Arab Texts into Latin. From the 13th century travel for the sake of knowledge was characteristic of both Arab and Jewish scholars, who wanted to study the religious and secular sciences and translations were being made from Greek, Arabic, Hebrew and other original texts into Latin.
Some names here would be unknown, Gilbert d' Aurillac of the Benedictine order and Jordanus de Nemore who taught algebra were significant, but Fibonacci should be familiar. Generally not understood is the role of the **Universities** in the revival of knowledge.
The university of Paris was founded in,1150 and Nicole Oresme, who became Bishop of Lisieux wrote on mathematics, physics, astronomy, and economics. Oresme's geometrical approach to the study of motion *The Latitude of Forms* appeared in in 1505 He gave an elegant proof of the important *Mean Speed Theorem.* (the Merton rule of uniform acceleration) that was used by Galileo later.

Renaissance Recovery and Innovation Chapter 10
The invention of Printing, in 1454, was responsible for the considerable variety of activities being disseminated across Europe, Pacioli produced his version of Bookkeeping, and we also begin to see the intricacies of that subject *algebra* with surprises in the stories of the solution of the **cubic and quartic** equations by del Ferro, Tartaglia, and Cardano, and some interesting detail on the nature of new 'numbers' that appear. Bombelli discovers that imaginary numbers come in conjugate pairs and also Viete with his algebra and spherical trigonometry is involved. Here the mathematics begins to get more complicated, possibly causing some problems for the reader.

Renaissance of Mathematics in Britain Chapter 11
(16th to mid 17th century)
Humanism came to England through the writings of Comenius and Erasmus who were the early champions of universal education, and Ramus insisted that teaching the Quadrivium subjects should be practical, methodical, and become relevant to daily life.
The accounts of the works of Recorde and Dee show that that the underlying Neo-Platonic philosophy was important in promoting the *usefulness* and the all-pervasive importance of mathematics in our lives. One important omission is the character of Thomas Digges, who was probably the most significant Mathematical Practitioner of this time. John Dee and Thomas Digges observed the Supernova of 1572 which persuaded them that Copernican theory was true.
Edmund Gunter**:** mathematician and well-known instrument maker. He was a prolific promoter of logarithms who invented a forerunner of the slide rule which became important for 'mechanising' the use of logarithm tables.
Thomas Harriot: astronomer, mathematician, ethnographer and translator. He was famous for his contribution to developing notation and structure in algebraic methods. His true contribution was not really understood until the work of Jackie Stedall (2003). And we have here John Napier and Henry Briggs; inventor and populariser of the system of logarithms.

This chapter is packed with information and ideas. It is accessible, and well-organised.

Astronomy Revolution Chapter 12
This chapter documents the central astronomical and cosmological concepts that were debated in this period and shows how they generated and were informed by the use of mathematics. Here there are important scientific issues, the stellar observations, the applications of mathematics to motion, falling bodies and acceleration that indicate the problems with traditional views, touching on the messy political and religious contexts.
The characters involved are Tycho Brahe with critical observations, Kepler, who worked out the planetary orbits, and Galileo who observed and calculated and convinced people of a rational, sun-centred universe declaring Copernicus was right.
This is a 'good read' with both popular stories and serious content

European Mathematics in the Early 17th century Chapter 13
Here we look at the works of Fermat, Pascal, and Descartes, as part of the network of communication of ideas facilitated by Mersenne, where the actors are learning to adapt old mathematics to new situations and problems, and invent new mathematics.
William Oughtred's *Clavis Mathematicae* (1631) broke from classical traditions setting out a more formal methodological and down-to-earth approach to learning, and was commended by many, both in England and on the continent.

Girard Desargues was an architect and the founder of Projective Geometry. The two pages here briefly describe his major theorems written in the early 17th century, lost, and rediscovered in 1864. Hopefully we will hear more from Desargues in Volume 2

Pierre de Fermat.
There is little here from his geometry which is much shortened from the six pages of discussion of maxima and minima and the area of the hyperbola in section 11 .C in F&G. The text here gives an overview and some examples of his number theory.

Rene Descartes
Here is a long section of about 24 pages on Descartes covering his rational philosophy, science and mathematics. His 'Discourse on Method' described and explained his philosophical views, and *La Geometrie,* as an appendix to the Discourse demonstrated how his method should work in mathematics. Descartes also describes Pappus' Locus problem and how his 'Method' can solve the general problem
This last section, as a description and critique of Descartes philosophy and method is well-structured, interesting, but quite difficult for the less experienced to read; the authors are expecting the reader to do some serious work here.

Concluding Remarks Chapter 14
From the introduction, we have:
"We do not assume knowledge of any specific piece of mathematics, but we assume some familiarity with the subject and a willingness to grapple with the details……
The book is aimed at the general mathematically inclined reader. We hope that it will provide a rich introduction not only to the history of mathematics, but to mathematics itself. The prerequisites gradually involve more mathematics, but we believe that each section of this book can be read as an introduction to the mathematics involved, as well as providing an absorbing account of history it describes". (pp. 2 – 3)

This final chapter acts as an overview, a reflection on the content and ambitions of the first thirteen Chapters. One can approach the context of historical accounts as parts of a dialogue: who were they writing to? What were they writing for (or about)? And there is the challenge to recognise the 'mathematician' as against the priest, teacher, clerk, or administrator.
Only after the rise of the universities do we get near to identifying the scholar as a *professional* scientist or mathematician.

The earliest form of writing that we might call mathematics was for public administration (Archaic Bookkeeping) predating perhaps the development of literature.
These people were responsible for running the economy, and of understanding the heavens. The role of Commentators, translators, and copiers bring out discrepancies, arguments and alternatives. In Greece biographies emerge but their reliability is dubious
Writings about all kinds of things appear; politics, architecture, philosophy.

Activities required months, years, and centuries of work to mature and test their reliability whether it was astronomical observation, or a method for calculation.

Conflict, exploitation and exploration open up new channels of communication, while pestilence changes populations and perspectives and opens up new possibilities and new affordances

Apart from the work of Gunter, development of instruments in Astronomy and Navigation with links to scientific institutions and independent activities are happening, but this is too much to include in this volume.

The authors give us this quotation from the 'father' of ancient studies:
Otto Neugebauer (1969: 71) in the context of paucity of contemporary sources:

"Of all the civilisations of antiquity, the Egyptian seems to me to have been the most pleasant. The excellent protection which the desert and sea provide for the Nile Valley prevented the excessive development of the spirit of heroism which have most often made life in Greece hell on earth."

However, in contrast he also wrote:
"The common belief that we gain "historical perspective" with increasing distance seems to me utterly to misrepresent the actual situation. What we gain is merely confidence in generalisations which we would never dare to make if we had access to the real wealth of contemporary evidence." (Brown University Press 1957 – preface to first edition)

We can still ask, "What (or who) is this book **for**"? The private scholar? The individual or college setting up a new course? Has the MAA plans for a new project? We will have to see how things develop.

But we must remember; this is not just a 'history' book. This book is a Resource. It describes a course that was written for an optional undergraduate mathematics programme. There are going to be challenges.

Highly recommended.

Leo Rogers.